Use of Recursive Wavelet Transform for Estimating Power System Frequency and Phasors

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Abstract— **Frequency, amplitude and phase angle are three major parameters describing a stationary sinusoidal signal. In power system, many applications such as the state monitoring and protective relaying measure these parameters as performance indices or critical variables in the algorithm criteria respectively. This paper proposes a new method for estimating the power system frequency and phasors in real time based on a recursive wavelet transform. The frequency estimation algorithm is capable of accurately estimating the frequency within three samples of an input signal. It features fast response and achieves accurate frequency estimation over a wide range of frequency changes. For estimating phasors, the signal sampling rate and data window length can be selected to meet desirable applications requirements such as high accuracy and low computational burden. Simulation results demonstrate that the proposed method achieves good performance.**

***Index Terms*—Power system frequency, phasor estimation, recursive wavelet transform (RWT), TVE (Total Vector Error)**

I. Introduction

IN power system, many real-time applications measure frequency and phasors of voltage and current for the purpose of monitoring, control or protection. Power system frequency as a key index of power quality can be indicative of system abnormal conditions and disturbances. The frequency and phasor parameters including amplitude and phase angle are critical variables used by the algorithms. How to rapidly and accurately estimate frequency and other phasor parameters is still a contemporary research topic of interest.

A variety of techniques for the real time estimation of power system frequency have been developed and evaluated in past two decades. As an example, Fourier algorithm, i.e. discrete Fourier transform (DFT) has been extensively applied to this field due to its low computation requirement. However, the implicit data window in DFT approach causes errors when frequency deviates from the nominal value [1]. To improve the performance of DFT based approaches, some adaptive methods based on feedback loop by turning the sampling interval [2], adjusting data window length [3], changing the nominal frequency used in DFT iteratively [1], and correcting the gains of orthogonal filters recursively [4] are proposed. Because of the inherent limitation in DFT, at least one cycle of analyzed signal is required, which hardly meets the demand of high-speed protection schemes. On the basis of stationary signal model, some non-linear curve fitting techniques, including extended Kalman filter [5] and recursive Least Squares algorithm [6], are adopted to estimate fundamental frequency. The accuracy is only reached in a narrow range around nominal frequency due to the truncation of Taylor series expansions of nonlinear terms. Some artificial intelligence techniques, such as genetic algorithm [7] and neural networks [8] have been used to achieve precise frequency estimation over a wide range with fast response. Although better performance can be achieved by these optimization techniques, the implementation algorithm is more complex and intensive in computation.

Conventional Fourier algorithm based phasor estimation approaches use fixed length of observation window and the resulting accuracy for a given signal is independent of sampling rate [9]. For some applications which require high speed response but moderately low accuracy, such as transmission line protection, or require high level accuracy but low output rate, such as calibration and testing the functions of intelligent electronic devices (IEDs) [10] and off-line power system disturbance analysis [11], those methods hardly satisfy the requirements.

Recursive wavelet approach has been introduced in protective relaying for a long time [12]-[14]. The improved model with single-direction recursive equations is more suitable for the application to real-time signal processing [13]. The band energy of any center frequency can be extracted through recursive wavelet transform (RWT) with moderately low computation burden.

RWT based power system frequency and phasor estimation method is proposed in this paper. The frequency estimation algorithm can produce the output using three continuous samples of an input signal, and the resulting accuracy is independent of the signal sampling rate. It responds quite fast although the time delay brought by pre-filters may be applied. The convergence analysis indicates that to achieve a certain level of accuracy measured as TVE (total vector error described in [15]), the higher sampling rate one uses, the shorter data window the computation needs, and vice versa. For example, to limit the error within 1% TVE, the algorithm converges within 0.5 cycle of input signal at 18 kHz sampling frequency.

The rest of the paper is organized as follows. Section II introduces the concept of wavelet, the recursive wavelet transform and its characteristics both in time and frequency

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domain. The frequency estimation algorithm is described in Section III. The methods for estimating phasor and analyzing the convergence characteristics are given in Section IV. Section V presents the details of performance evaluation. Conclusions are outlined at end.

II. Recursive Wavelet Transform

Mother wavelet function is defined as a function $\psi(t)$ which satisfies the admissibility condition:

$$C_{\Psi} = \int_{-\infty}^{+\infty} \frac{|\Psi(\omega)|^2}{|\omega|} d\omega < \infty$$

i.e. $\Psi(\omega)|_{\omega=0} = \int_{-\infty}^{+\infty} \psi(t) dt = 0$

where $\Psi(\omega)$ is the Fourier transform of $\psi(t)$.

A set of wavelet functions can be derived from $\psi(t)$ by dilating and shifting the mother wavelet, as given below:

$$\psi_{a,b}(t) = a^{-1/2} \cdot \psi(\frac{t-b}{a}), a > 0$$

where a and b are scaling (dilation) factor and time shifting (translation) factor, respectively.

A mother wavelet function is given as:

$$\psi(t) = (-\frac{\sigma^3 t^3}{3} - \frac{\sigma^4 t^4}{6} - \frac{\sigma^5 t^5}{15}) e^{(\sigma + j\omega_0)t} u(-t)$$

One can see that this wavelet is a complex function whose expression in frequency domain contains the real part and imaginary part. The Fourier transform is expressed as follows:

$$\Psi(\omega) = \left\{ \frac{6\sigma^5 - 2\sigma^3(\omega - \omega_0)^2}{[\sigma + j(\omega - \omega_0)]^6} \right\}^*$$

Setting $\sigma = 2\pi/\sqrt{3}$, $\omega_0 = 2\pi$ makes the function $\psi(t)$ admissible, i.e. $\Psi(\omega)|_{\omega=0} = 0$.

Fig. 1 and Fig. 2 give the time and frequency domain waveforms (real and imaginary parts) of $\psi(t)$ and $\Psi(\omega)$, respectively. The complex wavelet exhibits good time-frequency localization characteristics. The time-domain center t^* and radius Δt are -1.53 s and 0.48 s respectively. It features a band-pass filter, as shown in Fig. 2, with the frequency-domain center ω^* and radius $\Delta\omega$ are 6.28 (2π) rad and 1.09 rad. The quality factor Q, defined as the ratio of frequency center ω^* and band width $2\Delta\omega$, is constant $Q = \omega^*/2\Delta\omega = 2.87$.

To obtain the center frequency f_c of the band-pass filter, which is defined as the frequency in which the function reaches the maximum magnitude, we have the Fourier transform for the dilated wavelet function $\psi(t/a)$:

$$\Psi(a\omega) = \left\{ \frac{6\sigma^5 - 2\sigma^3(a\omega - \omega_0)^2}{[\sigma + j(a\omega - \omega_0)]^6} \right\}^*$$

$|\Psi(a\omega)|$ reaches the maximum value when $a \cdot \omega = \omega_0$, i.e. $a \cdot 2\pi \cdot f_c = 2\pi$. Thus, we have $f_c = 1/a$. That is the scale factor a is reciprocal to the center frequency f_c of the band-pass filter.

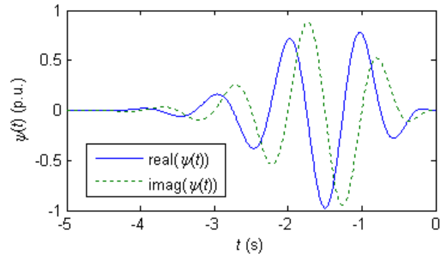

Fig. 1 Time domain waveforms of $\psi(t)$

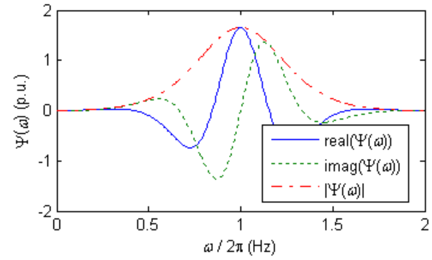

Fig. 2 Frequency domain waveforms of $\Psi(\omega)$

The wavelet transform coefficient in scale a for a given signal $x(t)$ can be expressed as below:

$$W_{x(t)}(a,b) = a^{-1/2} \int_{-\infty}^{+\infty} x(t) \cdot \psi(\frac{t-b}{a})^* dt, t > 0$$

(1)

Let ΔT be the sampling period, thus $t = k\Delta T$. Then we have the discrete wavelet transform for the signal $x(k\Delta T)$ [13]:

$$\begin{aligned} W_{x(k\Delta T)}(f,k) &= \Delta T \sqrt{f} \{\lambda_1 x(k-1) + \lambda_2 x(k-2) \\ &+ \lambda_3 x(k-3) + \lambda_4 x(k-4) + \lambda_5 x(k-5)\} \\ &- \beta_1 W_{x(k\Delta T)}(f,k-1) - \beta_2 W_{x(k\Delta T)}(f,k-2) \\ &- \beta_3 W_{x(k\Delta T)}(f,k-3) - \beta_4 W_{x(k\Delta T)}(f,k-4) \\ &- \beta_5 W_{x(k\Delta T)}(f,k-5) - \beta_6 W_{x(k\Delta T)}(f,k-6) \end{aligned} \tag{2}$$

where,

$\alpha = e^{-f\Delta T(\sigma - j\omega_0)}$

$\lambda_1 = \alpha[(\sigma f\Delta T)^3/3 - (\sigma f\Delta T)^4/6 + (\sigma f\Delta T)^5/15]$

$\lambda_2 = \alpha^2[2(\sigma f\Delta T)^3/3 - 5(\sigma f\Delta T)^4/3 + 26(\sigma f\Delta T)^5/15]$

$\lambda_3 = \alpha^3[-6(\sigma f\Delta T)^3/3 + 22(\sigma f\Delta T)^5/5]$

$\lambda_4 = \alpha^4 2(\sigma f\Delta T)^3/3 + 5(\sigma f\Delta T)^4/3 + 26(\sigma f\Delta T)^5/15$

$\lambda_5 = \alpha^5[(\sigma f\Delta T)^3/3 + (\sigma f\Delta T)^4/6 + (\sigma f\Delta T)^5/15]$

$\beta_1 = -6\alpha, \beta_2 = 15\alpha^2, \beta_3 = -20\alpha^3$

$\beta_4 = 15\alpha^4, \beta_5 = -6\alpha^5, \beta_6 = \alpha^6$

In formula (2), f represents the center frequency which is reciprocal to the scale factor a. To extract the frequency band energy centered in 60 Hz, for instance, simply apply $f = 60$ to (2). One can notice that wavelet transform coefficients can be calculated recursively with the historical data. Thus, this type of transform is so-called the recursive wavelet transform (RWT).

III. Frequency Estimation

The recursive wavelet (RW) features a complex wavelet whose wavelet coefficients (real part and imaginary part) contain the phase information of the input signal, based on which the algorithm for estimating the power system frequency is derived.

Consider a sinusoidal signal expressed in complex form:

$$x(t) = A_m e^{j(\omega t+\varphi)} \quad t \geq 0$$

(3)

where A_m is the amplitude, φ is the phase angle and $\omega = 2\pi \cdot f$.

Apply RWT to signal $x(t)$ in scale a using (1). As derived in Appendix, we obtain,

$$W_{x(t)}(a,b) = A_m e^{j(\omega b+\varphi)} \cdot E(a,b,\omega) \quad b \geq 0$$

One can see that $W_{x(t)}(a, b)$ contains the same expression of input signal as (3), as well as the frequency information in coefficient $E(a, b, \omega)$. Thus E can be obtained by dividing $x(b)$ from $W_{x(t)}$, i.e. $E = W_{x(t)} / x(b)$. Calculate the derivative of E with respect to b, we have,

$$|E'| e^{j\theta(b)} = e^{j(\omega_0 \cdot b/a - b\cdot\omega)} \cdot F(a,b)$$

where $F(a,b) = a^{1/2} \cdot (\frac{\sigma^3 b^3}{3a^3} - \frac{\sigma^4 b^4}{6a^4} + \frac{\sigma^5 b^5}{15a^5}) \cdot e^{-\sigma \cdot b/a}$

Apparently, F is real number. Thus,

$$\theta(b) = \omega_0 \cdot b/a - b\cdot\omega$$

Differentiate θ with respect to b, we obtain,

$$\omega = \omega_0 / a - \theta'$$

i.e. $f = (\omega_0 / a - \theta')/2\pi$

Assume the sampling period ΔT, then formulas for E, θ and f can be written in discrete forms:

$$E(k) = W_{x(k\Delta T)}(k) / x(k) \tag{4}$$

$$\theta(k) = \tan^{-1}(\frac{E(k+1) - E(k)}{\Delta T}) \tag{5}$$

$$f(k) = (\frac{\omega_0}{a} - \frac{\theta(k+1) - \theta(k)}{\Delta T})/2\pi \tag{6}$$

Based on formulae (4), (5) and (6), we can conclude that to calculate the frequency only three continuous samples of input signal $x(k)$ are needed. Hence the data window length for estimating the frequency of a sinusoidal signal is three times the sampling period ($3\Delta T$). Furthermore, the sampling rate of signal being analyzed has no impact on the method accuracy. Flow chart in Fig. 3 illustrates the implementation procedures of the proposed frequency estimation approach. It should be noted that the frequency f (corresponding to the scale factor a) can be initialized with any value when calculating RWT coefficients $W(k)$ using (2). Estimation result is independent of the initial value, thus we pick fundamental frequency 60 Hz (or 50 Hz) as the initial value in power system applications.

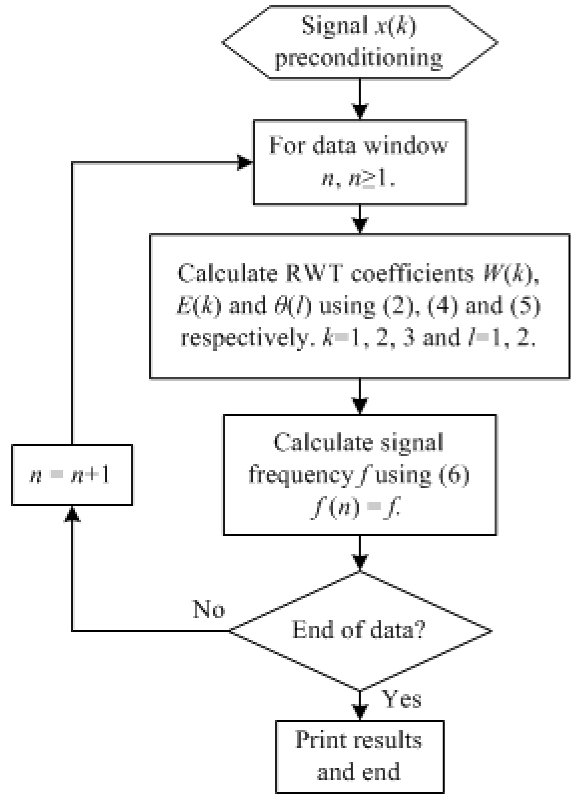

Fig. 3 Flow chart of the frequency estimation

IV. Phasor Estimation

A. RWT Based Phasor Estimation Method

Use the same sinusoidal signal model as expressed in (3), and apply RWT to $x(t)$ in scale a using (1). Derivation of the wavelet transform coefficient is given in the Appendix. We have,

$$W_{x(t)}(a,b) = A_m e^{j(\omega b+\varphi)} \cdot G(a,b)$$

One should note that the coefficients E and G have the same expression except that the signal frequency ω is known variable to G.

Dividing $W_{x(t)}$ by G we have,

$$A_m e^{j(\omega b+\varphi)} = |W_{x(t)}(a,b)/G(a,b)| e^{j\theta(a,b)}$$

Thus, the formulae for computing the amplitude A_m and phase angle φ are:

$$A_m(a,b) = |W_{x(t)}(a,b)/G(a,b)|$$

$$\varphi(a,b) = \theta(a,b) - \omega b$$

where a, ω are constant, and b is time variable.

Discrete forms can be derived by introducing the sampling period ΔT. Assume the sampling frequency rate is N_s times of the signal frequency f, i.e. $f = 1/(N_s \cdot \Delta T)$.

$$A_m(f,k) = |W_{x(k\Delta T)}(f,k)/G(f,k)| \tag{7}$$

$$\varphi(f,k) = \theta(f,k) - 2\pi \cdot k / N_s \tag{8}$$

where $W_{x(k\Delta T)}(f,k)$ is the discrete IRW coefficient in scale f calculated using (2). $G(f,k)$ is a constant vector whose expression is given in the Appendix.

B. Analysis of the Convergence Characteristics

Theoretically, the phasor (amplitude and phase angle) of a sinusoidal signal can be accurately calculated using formula (7) and (8) with two samples, i.e. the algorithm converges to the real value within two samples. Then the length of the data window is $2\Delta T$, and the result is independent from the signal sampling rate. The studies indicate that both sampling rate and window length affect the convergence characteristics because of two factors. One is that formula (7) and (8) are derived based on the assumption that the error resulting from the discrete computation is negligible. Another is the error introduced by the inherent settling process in recursive equations.

To analyze the convergence characteristics, we define the window length l_s as the cycle of the input signal, which is independent of the signal sampling rate f_s defined as N_s times of signal frequency f in Hz. Apparently l_s and f_s determine the number of samples N within a data window, i.e. $N = l_s{\cdot}f_s / f = l_s{\cdot}N_s$. Total vector error (TVE) is used to measure the phasor accuracy. Once the amplitude error ΔA_m (in percent of real value) and the phase error $\Delta\varphi$ (in degrees) are available, the expression for TVE is given by $\mathrm{TVE} = \sqrt{(\Delta v)^2 + (\Delta\theta / 0.573)^2}$, where 0.573 is the arcsine of 1% in degree.

Relationship between TVE and two variables l_s and f_s is illustrated in Fig. 4, in which the signal sampling frequency is simulated from 3 kHz to 24 kHz while the window length is from 0.5 cycle to 2 cycles. To make the result less than certain accuracy, for example 1% TVE, the sampling frequency must be 9 kHz or higher once the window length is fixed at 1 cycle, and the data window can be shortened to 0.5 cycle once the sampling rate is 18 kHz or higher.

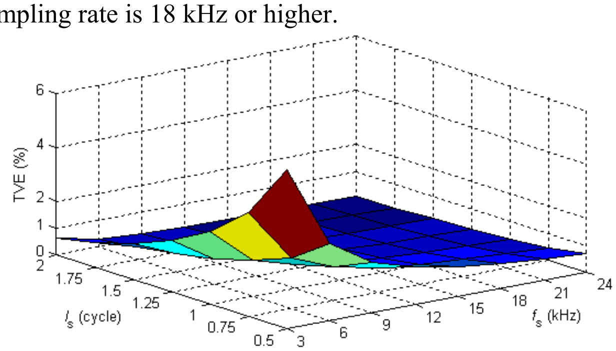

Fig. 4 Convergence characteristics of phasor estimation

As we know the signal sampling rate is relevant to the hardware cost while the window length determines the time interval for the first phasor calculation outcome. RWT based phasor estimation method provides the feature that the sampling rate and data window length can be selected in terms of the applications requirements. Besides, the result accuracy is predictable. Fig. 5 gives the flow chart of the implementation procedures.

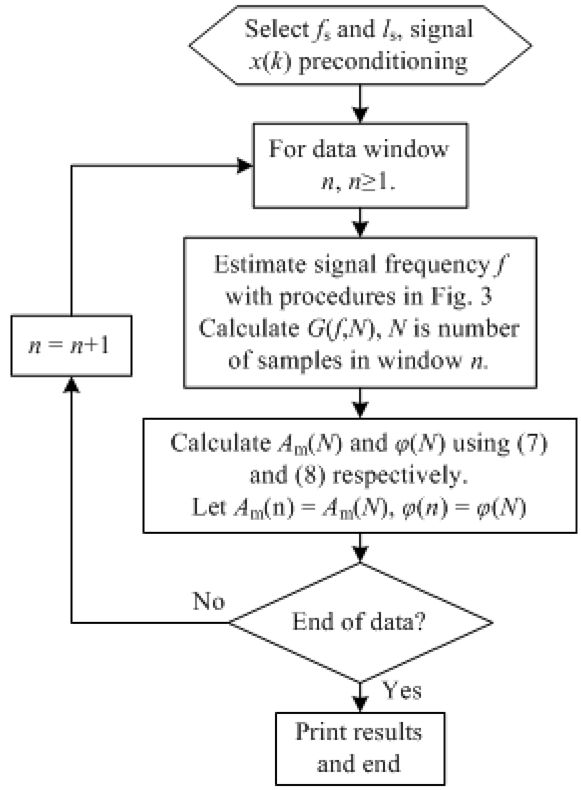

Fig. 5 Flow chart of the phasor estimation

In practice, as shown in Fig. 5 for a given data window with N samples, vector $W_{x(k)}$ (k=1, 2, …, N) can be calculated using recursive equation (2). Coefficient G at the last point N, i.e. $G(f,N)$, can be computed once signal frequency f is estimated. Then, $A_m(N)$ and $\varphi(N)$ for the given data window can be calculated using $W_{x(k)}(N)$ and $G(f,N)$. As one can see the computation burden is fairly low. It can satisfy the time response requirement of time-critical applications.

V. Performance Evaluation

In this section, performances of the frequency and phasor estimation algorithm are evaluated with a stationary sinusoidal signal model.

A. Frequency Estimation

A stationary signal model with constant frequency given in (9) is used for testing the frequency estimation algorithm. In simulation, we assign A_m = 1 p.u., φ = 60º, sampling rate N_s = 50, i.e. f_s = 3 kHz.

$$x(t) = A_m e^{j(2\pi\cdot f\cdot t+\varphi)} \tag{9}$$

For the given signal, f varies from 40 Hz to 70 Hz in 2 Hz steps. Fig. 6 depicts the results, in which f_e represents the estimated frequency. Table I gives some of the simulation results. We can see that the estimation errors, defined as $Err = \frac{|f_e - f|}{f} \times 100\%$, are zeros for the given signal model.

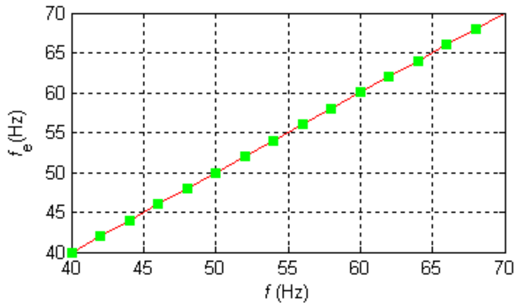

Fig. 6 Test results for frequency estimation

TABLE I
TEST RESULTS FOR FREQUENCY ESTIMATION

f (Hz)	40	46	50	56	60	66	70
f_e (Hz)	40.00	46.00	50.00	56.00	60.00	66.00	70.00
Err (%)	0.00	0.00	0.00	0.00	0.00	0.00	0.00

B. Phasor Estimation

The signal model in (9) is used to evaluate the performance of the phasor estimation algorithm. In simulation, signal frequency f is constant assigned with fundamental frequency 60 Hz, the window length l_s = 0.5 cycle. Referring to the Fig. 4, we select sampling frequency f_s = 18 kHz to guarantee the error measured with TVE below 1%.

Tests are performed with the amplitude A_m varying from 0.4 to 1.4 in 0.1 step and the phase angle φ varying from 0º to 180º in 20º steps. Find the maximum TVE for each case and record it as the error for the corresponding case. Test results are given in Fig. 7. For the given signal model with its parameters varying over broad range, phasors can be calculate in 0.5 cycle and the errors can be limited within 1% TVE.

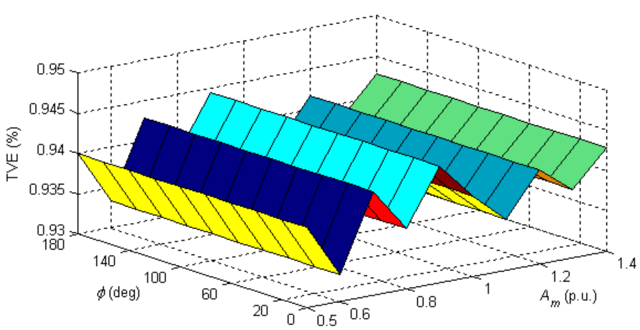

Fig. 7 Test results for phasor estimation

VI. CONCLUSIONS

A new method for estimating in real-time power system frequency and phasor based on recursive wavelet transform is proposed. The frequency estimation algorithm features rapid response and accurate result over a wide range of frequency deviations. It uses only three subsequent samples for outputting frequency calculation result for a stationary sinusoidal signal. For estimating phasor, sampling rate and observation window length can be chosen to meet selected applications requirements. Analysis of the algorithm convergence characteristics indicates that to achieve a certain level of accuracy measured as TVE, the higher the sampling rate one uses, the shorter the computation data window and the faster the rate the method outputs phasor, and vice versa. Performance evaluation shows that the proposed approach is capable of outputting precise frequency over a broad range. Selecting sampling frequency at 18 kHz, phasor can be computed within 0.5 cycle of input signal and the error is limited within 1% TVE. Besides, the computation requirement is fairly low, thus it can satisfy the time critical demand of the high speed protection schemes.

VII. APPENDIX

The RWT coefficient of a given signal $x(t)$ is:

$$W_{x(t)}(a,b) = a^{-1/2}\int_0^b x(t)\cdot\psi^*(\frac{t-b}{a})dt \quad b\ge 0$$

$$= a^{-1/2}\int_0^b A_m e^{j(\omega t+\varphi)}\cdot[-\frac{\sigma^3}{3}(\frac{t-b}{a})^3-\frac{\sigma^4}{6}(\frac{t-b}{a})^4$$

$$-\frac{\sigma^5}{15}(\frac{t-b}{a})^5]e^{(\sigma-j\omega_0)\cdot(\frac{t-b}{a})}dt$$

Denote $t = l\cdot a+b, l\in[-b/a,0]$, we have,

$$W_{x(t)}(a,b) = A_m e^{j(\omega b+\varphi)}\cdot E(a,b,\omega)$$

where

$$E(a,b,\omega)=\sqrt{a}\int_{-\frac{b}{a}}^{0} e^{[\sigma+j(a\omega-\omega_0)]l}\cdot(-\frac{\sigma^3}{3}l^3-\frac{\sigma^4}{6}l^4-\frac{\sigma^5}{15}l^5)dl$$

The RWT coefficient of the signal $x(t)$ is:

$$W_{x(t)}(a,b) = a^{-1/2}\int_0^b x(t)\cdot\psi^*(\frac{t-b}{a})dt \quad b\ge 0$$

$$= a^{-1/2}\int_0^b A_m e^{j(\omega t+\varphi)}\cdot[-\frac{\sigma^3}{3}(\frac{t-b}{a})^3-\frac{\sigma^4}{6}(\frac{t-b}{a})^4$$

$$-\frac{\sigma^5}{15}(\frac{t-b}{a})^5]e^{(\sigma-j\omega_0)\cdot(\frac{t-b}{a})}dt$$

Denote $t = l\cdot a+b, l\in[-b/a,0]$ and

$\eta=\sigma+j(a\omega-\omega_0)$, we have,

$$W_{x(t)}(a,b) = A_m e^{j(\omega b+\varphi)}\cdot G(a,b)$$

where

$$G(a,b)=\sqrt{a}\int_{-\frac{b}{a}}^{0} e^{[\sigma+j(a\omega-\omega_0)]l}\cdot(-\frac{\sigma^3}{3}l^3-\frac{\sigma^4}{6}l^4-\frac{\sigma^5}{15}l^5)dl$$

$$=\sqrt{a}\cdot\{\frac{-\sigma^3}{3}\cdot[-\frac{(\frac{-b}{a})^3}{\eta}\cdot e^{\eta\cdot(\frac{-b}{a})}+\frac{3(\frac{-b}{a})^2}{\eta^2}$$

$$\cdot e^{\eta\cdot(\frac{-b}{a})}-\frac{6(\frac{-b}{a})}{\eta^3}\cdot e^{\eta\cdot(\frac{-b}{a})}-\frac{6}{\eta^4}\cdot(1-e^{\eta\cdot(\frac{-b}{a})})]$$

$$-\frac{\sigma^4}{6}\cdot[-\frac{(\frac{-b}{a})^4}{\eta}\cdot e^{\eta\cdot(\frac{-b}{a})}+\frac{4(\frac{-b}{a})^3}{\eta^2}\cdot e^{\eta\cdot(\frac{-b}{a})}$$

$$-\frac{12(\frac{-b}{a})^2}{\eta^3}\cdot e^{\eta\cdot(\frac{-b}{a})}+\frac{24(\frac{-b}{a})}{\eta^4}\cdot e^{\eta\cdot(\frac{-b}{a})}$$

$$+\frac{24}{\eta^5}\cdot(1-e^{\eta\cdot(\frac{-b}{a})})]-\frac{\sigma^5}{15}\cdot[-\frac{(\frac{-b}{a})^5}{\eta}\cdot e^{\eta\cdot(\frac{-b}{a})}$$

$$+\frac{5(\frac{-b}{a})^4}{\eta^2}\cdot e^{\eta\cdot(\frac{-b}{a})}-\frac{20(\frac{-b}{a})^3}{\eta^3}\cdot e^{\eta\cdot(\frac{-b}{a})}$$

$$+\frac{60(\frac{-b}{a})^2}{\eta^4}\cdot e^{\eta\cdot(\frac{-b}{a})}-\frac{120(\frac{-b}{a})}{\eta^5}\cdot e^{\eta\cdot(\frac{-b}{a})}-\frac{120}{\eta^6}\cdot(1-e^{\eta\cdot(\frac{-b}{a})})]\}$$

The discrete form $G(f,k)$ has the same expression with $a=1/f$ and $b=k\Delta T$ instead.

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IX. Biographies

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Mladen Kezunovic (S'77, M'80, SM'85, F'99) received the Dipl. Ing., M.S. and Ph.D. degrees in electrical engineering in 1974, 1977 and 1980, respectively. Currently, he is the Eugene E. Webb Professor and Site Director of Power Engineering Research Center (PSerc), an NSF I/UCRC.at Texas A&M University. He worked for Westinghouse Electric Corp., Pittsburgh, PA, 1979-1980 and the Energoinvest Company, in Europe 1980-1986, and spent a sabbatical at EdF in Clamart 1999-2000. He was also a Visiting Professor at Washington State University, Pullman, 1986-1987 and The University of Hong Kong, fall of 2007. His main research interests are digital simulators and simulation methods for relay testing as well as application of intelligent methods to power system monitoring, control, and protection. Dr. Kezunovic is a Fellow of the IEEE, member of CIGRE and Registered Professional Engineer in Texas.