Controlled Islanding to Prevent Cascade Outages Using Constrained Spectral k-Embedded Clustering

Ahad Esmaeilian, *Student Member, IEEE*, and Mladen Kezunovic, *Fellow, IEEE*
Department of Electrical and Computer Engineering
Texas A&M University
College Station, Texas, USA
ahadesmaeilian@tamu.edu, kezunov@ece.tamu.edu

***Abstract*— Controlled islanding scheme is frequently considered as the final solution to retain power system operation under cascading event outages which can lead to a major blackout. The islanding scheme can improve the power system transient stability by preserving stable areas from further outages as well as reducing the complexity and overall time required for restoration process. In this paper a constrained spectral k-embedded clustering method is defined to determine an islanding scheme with minimal active power flow disruption, while addressing generator's coherency problem. The proposed spectral clustering method uses k-medoids algorithm which is more accurate and computationally efficient than conventional k-means algorithm. The proposed scheme is tested using the IEEE 9-, IEEE39- and IEEE 118-bus test systems. Simulation results show effectiveness of the proposed methodology in creation of stable islands and prevention of system blackouts.**

***Index Terms*— Cascading outage, controlled islanding, constrained spectral clustering, graph theory.**

I. INTRODUCTION

Power system blackout is a fairly complicated phenomenon with very low expectation of occurrence but potentially devastating social and economic impacts. For instance, on August 14th 2003, a large blackout affected the North East of the USA and Canada with around 50 million people, contributed to at least 11 deaths and estimated total costs between 4 to 10 billion dollars [1]. Analysis of recent historical blackouts has shown that the grid catastrophic events occurred following a series of cascading events such as transmission line outages, overloads and malfunctions of protective relays [2-4]. After several disturbances take place in power grid, the overall system operates closer to its stability margin. Intentional islanding can effectively interrupts sequence of cascade outages and prevents large area blackouts. It is proven that intelligent separation of power system into islands at the primary stage of cascade outages can reduce or even eliminates risk of major blackouts [5].

The intentional islanding also known as "controlled islanding" is generally classified as an optimization problem which can be categorized into the following two main groups. The first group is defined based on the concept of slow coherency grouping of generators [6-10]. These algorithms solve max-flow min-cut problem along with the additional knowledge of location of disturbance to determine the minimum cutsets among the coherent groups of generators. The slow coherency grouping is based on a linearized electromechanical model of the system. Therefore, it may fail in determining appropriate islands due to highly nonlinear nature of real size power systems.

The second group determines islands considering graph search methods such as ordered binary decision diagrams (OBDD) [11, 12]. First, the proposed method simplifies the original power system using an effective graph theory based algorithm. Then, it narrows down the solution space using OBDDs to satisfy the "equality" constraints for controlled islanding. Finally, it uses power-flow results to determine if any of the strategies satisfy "inequality" constraints. The major drawback of OBDD searching methods is that the simplification of the original power system may result in losing superior solutions.

In the recent years, several researchers proposed controlled islanding based on graph clustering algorithms [13, 14]. In [13], a k-way spectral clustering algorithm is presented which splits the power system into islands based on minimum load-generation imbalance. In [14], a multi-level kernel k-means method is proposed to create islands considering minimum power flow disruption. While these two methods are computationally efficient, dynamic constraints were not considered in any of them. Neglecting generator behavior may split power system into instable islands.

In [15], an algorithm based on angle modulated particle swarm optimization is presented which utilizes minimal power imbalance to achieve a controlled islanding solution. The computational efficiency of this method is enhanced by neglecting the connectivity of the sub-graphs while it makes the islanding solution vulnerable to containing isolated buses.

In this paper, a controlled islanding method based on constrained spectral k-embedded clustering is presented which offers high speed performance without need to reduce the size of power system model. The algorithm splits power system into islands by constructing the graph of the system whose nodes are weighed based on minimal power-flow disruption. To assure the dynamic stability of each island, the generator coherency information is considered as constraints of the spectral k-embedded clustering problem.

The rest of the paper is organized as follows. Section II introduces the spectral clustering theory. Section III describes the proposed controlled islanding scheme. In section IV, the proposed method is applied to IEEE 118 bus system and the results have been discussed. Finally, section V concludes with listing the main contribution of the paper.

II. BACKGROUND THEORY

A. Graphs Representation of Power System

Any power system can be mapped into an undirected graph where the graph's nodes (vertices) and links (edges) represent buses and lines, respectively. Since the vertices and edges of the graph cannot give the information about the power grid, edges should be weighted with some meaningful parameters. Following, two popular parameters are mentioned as edge's weight.

Admittance of the lines which can measure the electrical distance between vertices can be used as edge weigth.

$$W = \begin{cases} w_{ij} = |Y_{ij}| & i \neq j \\ w_{ij} = 0 & i = j \end{cases} \tag{1}$$

The edge weighting based on admittance matrix will result in a so called "static" weight which means it will remain the same for a given network.

Power flow across the lines can be used as the edge weigth to determine lines importance in a given operating condition.

$$W = \begin{cases} w_{ij} = \dfrac{|P_{ij}| + |P_{ji}|}{2} & i \neq j \\ w_{ij} = 0 & i = j \end{cases} \tag{2}$$

Using average power flow can compensate lines' losses. The edge weighting based on power flow will result in "dynamic" weight since power flow changes according to actual operating conditions.

Once the edge matrix has been built, the un-normalized Laplacian of the graph can be defined as:

$$L = \begin{cases} -w_{ij} & i \neq j \\ d_i & i = j \end{cases}$$
$$d_i = \sum_{j=1}^{n} w_{ij} \tag{3}$$

The matrix L can be written as $L = D - W$, where D is a diagonal matrix with nonzero entries d_i. Laplacian matrix of a graph G has distinct properties which can be used to perform spectral clustering as it will be discussed in the next subsection.

B. Spectral k-Embedded Clustering

Clustering is a procedure of classifying objects into groups whose members are analogous in some way. Spectral clustering is defined as the process of determining groups of vertices (clusters) which are strongly linked to each other but loosely connected to vertices in other clusters utilizing Laplacian eigenvalues and eigenvectors. The controlled islanding can be described as a graph min-cut problem which splits the graph G into sub-graphs using spectral clustering. The spectral clustering can be summarized into following steps:

- Building Laplacian matrix L of graph G
- Solving the generalized Eigenvalue problem $Lv = \lambda Dv$
- Selecting k eigenvectors ($v_1, ..., v_k$) corresponding to the k lowest eigenvalues to define a k-dimensional subspace.
- Defining $J \in R^{n \times k}$ as the matrix containing vectors $v_1, ..., v_k$ as columns.
- Defining $y_i \in R^k$ as the vector corresponding to the ith row of J.
- Clustering the nodes $y_i \in R^k$ using some standard clustering algorithm such as k-means, k-medoids, etc.

It should be noted that un-normalized spectral clustering which can be obtained by calculating eigenvectors of L often results in clusters consists of single vertex. To avoid such clusters, normalized spectral clustering can be deployed by solving generalized Eigenvalue problem ($Lv = \lambda Dv$).

III. CONTROLLED ISLANDING SCHEME

After several disturbances occurred in power system, the system gradually drifts to instability region. In such cases, the primary control actions may fail to restore the system in a timely manner. Consequently, the power system may enter the stage of fast cascading outages which will result in large-area blackouts. In this condition, the controlled islanding scheme could be deployed as a final solution to prevent cascade outages and major blackouts. An advanced controlled islanding strategy should determine the proper switching sequences to create islands which can satisfy both the dynamic and static stability constraints. These constraints are discussed in next subsections.

A. Dynamic Constraints

A severe disturbance in a power network can initiate electromechanical wave oscillations which may cause generators to lose their synchronism. In this situation, generators with strong dynamic coupling swing together and are called coherent generators, whereas generators with weak dynamic coupling swing against each other [16, 17]. An islanding solution should be formed in such a way that coherent generators must remain in same islands to improve the transient stability and decrease the possibility of further outages. In this paper, we assumed that the generator coherency information is available and obtained from approaches such as those described in [18, 19]. This information is used as input to the proposed controlled islanding scheme to construct the constraint matrix which will be introduced in section III.C.

B. Static Constraints

To achieve a proper control islanding strategy, securing the transient stability is more critical than load-generation balance. Since, an island without load-generation balance can be preserved with partial load shedding while an island with negative stability margin will collapse even though a good load-generation balance is secured in it. The two mainly used objective functions to achieve control islanding solution are defined as follows. First, objective function is defined based on the minimal power flow imbalance which secures islands with good load-generation balance. Second, objective function is defined based on minimal power flow disruption which secures minimum change on transmission lines power flow pattern compare to pre-disturbance situation. The latter objective function which is used in this paper reduces overloading problem of transmission lines within the islands, while it enhances the stability of the islands. Consequently, it decreases the complexity and facilitates reconnection of islands to each other.

C. Constrained Spectral k-Embedded Clustering

As mentioned earlier, the controlled islanding problem can be evaluated as a graph-cut problem using constrained spectral clustering. In the constrained spectral clustering two types of constraints can be defined as Must Link (ML) and Cannot Link (CL) [20, 21]. A ML constraint between two vertices assures that those vertices will be in the same cluster. However, a CL constraint guarantees that the vertices will be in different clusters. To apply constraints into spectral clustering problem, constraint matrix Q is defined as below:

$$Q = \begin{cases} -1 & if \quad (x_i, x_j) \in CL \\ +1 & if \quad (x_i, x_j) \in ML \\ 0 & else \end{cases} \tag{4}$$

To determine if the constraints are properly attained by the clustering solution, the following index is defined:

$$u^T Q u = \sum_{i,j} u_i u_j q_{ij} \tag{5}$$

where $u \in \{-1, +1\}^N$ is called cluster indicator vector. The above encoding scheme can be further extended by relaxing u and Q such that:

$$u \in R^N \ , \ Q \in R^{N \times N} \tag{6}$$

If nodes i and j are related to the same cluster then $Q_{ij} > 0$; if nodes i and j are related to separate clusters then $Q_{ij} < 0$. The greater value of $u^T Q u$ confirms that constraints defined by Q are better satisfied. The constraint matrix Q and the Laplacian matrix L can be converted to their normalized form as follows.

$$\begin{aligned} L_N &= D^{-1/2} L D^{-1/2} \\ Q_N &= D^{-1/2} Q D^{-1/2} \end{aligned} \tag{7}$$

where D is diagonal matrix introduced in section II.A.

Then, the constrained spectral clustering can be defined as a constrained optimization problem with the following definition [21]:

$$\begin{aligned} &\arg\min \quad v^T L_N v \\ &s.t. \quad v^T Q_N v > \beta, \ v^T v = vol, \ v \neq D^{1/2} 1 \end{aligned} \tag{8}$$

where $vol = \sum_{i=1}^{N} d_{ii}$ is the volume of the graph and β is the threshold value defined to confirm the constraints satisfaction. $v^T L_N v$ is the cost of the cut, $v^T v = vol$ is defined to normalize v and $v \neq D^{1/2} 1$ rules out the trivial solution.

The objective function in (8) can be solved using Karush-Kuhn-Tucker Theorem [22]. After a few mathematical steps described in [21], optimal solution of (8) could be obtained by solving the generalized eigenvalue problem as below.

$$L_N v = \lambda \left(Q_N - \frac{\beta}{vol} I \right) v \tag{9}$$

Considering spectral *k*-Embedded clustering explained in section II.B along with generator coherency information, the proposed method can be implemented within following steps.

- Defining k (total number of clusters) based on generator coherency information.
- Computing edge weight matrix using (2) as well as normalized Laplacian matrix L_N using (3) and (7).
- Computing constraint matrix Q using generator coherency information.
- Solving the generalized eigenvalue system in (9)
- Removing eigenvectors associated with zero or negative eigenvalues.
- Normalizing the remaining eigenvectors $v \leftarrow \frac{v}{\|v\|} \sqrt{vol}$.
- Selecting k-1 eigenvectors ($v_1, ..., v_{k-1}$) related to the k-1 lowest eigenvalues.
- Defining $V^* \leftarrow \arg\min V^T L_N V$ where $V \in R^{n \times k-1}$ is the matrix containing vectors $v_1, ..., v_{k-1}$ as columns.
- Clustering nodes using k-medoids algorithm [23].

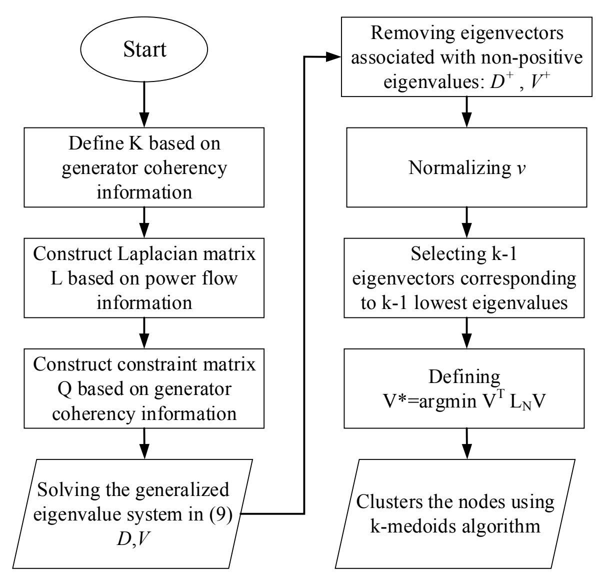

Figure 1. Constrained spectral k-embedded clustering scheme

It should be noted that using k-embedded spectral clustering instead of bisection spectral clustering [24] can improve partitioning process by giving more flexibility in adjusting the clusters. In addition, clustering based on k-medoids algorithm can avoid noise and data anomalies comparing to k-means due to minimizing sum of variances instead of squared distances among data-points.

IV. Test Results

In this section, the proposed method is tested using IEEE 9-bus, IEEE 39-bus and IEEE 118-bus test systems [25-27]. The controlled islanding scheme is developed in MATLAB. As described below, for each test system, the controlled islanding method has been tested and an islanding solution is suggested.

A. IEEE 9-Bus System

Fig. 2 shows the single-line diagram of IEEE 9-bus system. The two coherent generator groups are determined in [24] as {G1} and {G2, G3}. In Fig. 2, the value assigned to each line (edge weight) presents the per unit average active power flow obtained from powerflow results.

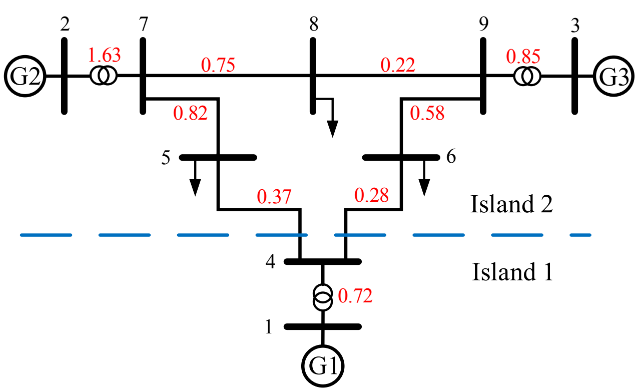

Figure 2. IEEE 9-bus test system

Considering Fig. 2 as graph G where the buses and lines are vertices and edges, respectively, with the edge weight labeled at each line, the Laplacian matrix can be constructed as follows.

$$L=\begin{bmatrix} 0.72 & 0 & 0 & -0.72 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.63 & 0 & 0 & 0 & 0 & -1.63 & 0 & 0 \\ 0 & 0 & 0.85 & 0 & 0 & 0 & 0 & 0 & -0.85 \\ -0.72 & 0 & 0 & 1.37 & -0.37 & -0.28 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.37 & 1.19 & 0 & -0.82 & 0 & 0 \\ 0 & 0 & 0 & -0.28 & 0 & 0.86 & 0 & 0 & -0.58 \\ 0 & -1.63 & 0 & 0 & -0.82 & 0 & 3.2 & -0.75 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.75 & 0.97 & -0.22 \\ 0 & 0 & -0.85 & 0 & 0 & -0.58 & 0 & -0.22 & 1.65 \end{bmatrix} \tag{10}$$

Then, the generator coherency information is used to construct the constraint matrix Q as defined in (4). The constraint matrix Q represents the ML constraint between G2 and G3, and CL constraint between G1 and {G2 and G3}.

$$Q=\begin{bmatrix} 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{11}$$

Next, by applying steps 3 to 8 of the proposed method, a single cutset is obtained as depicted with dashed line in Fig. 2 with the cut of 0.65 p.u. The two islands include buses {1, 4} and {2, 3, 5, 6, 7, 8, 9}, respectively. As expected, the suggested islanding solution satisfies generator coherency constraints. In addition, the load-generation balance could be preserved without any need of load shedding. As it can be seen in Table I, the active and reactive loads in both islands are less than the active and reactive generation capacity.

TABLE I. IEEE 9-Bus System Load-Generation Balance

Island no.	Active Load (P_L)	Active Gen Capacity (P_G)	Reactive Load (Q_L)	Reactive Gen Capacity (Q_G)
1	0*	2.5	0.28	±3
2	3.15	5.5	1.15	±6

*Values are in p.u. base of 100MVA.

B. IEEE 39-Bus System

The single-line diagram of IEEE 39-bus system is shown in Fig. 3. The three coherent generator groups are determined in [24] as {G1}, {G2, G3, G4, G5, G6, G7} and {G8, G9, G10}. By applying the proposed method to the IEEE 39-bus test system, three islands are obtained as depicted with dashed lines in Fig. 3. The suggested cutset includes lines between buses {1-2, 3-4, 3-18, 8-9, 17-27} with the total cut of 3.1 p.u. As expected, the suggested islanding solution satisfies generator coherency constraints. In addition, the load-generation balance is preserved in island 3. As it can be seen in Table II, the reactive loads in all islands are less than the

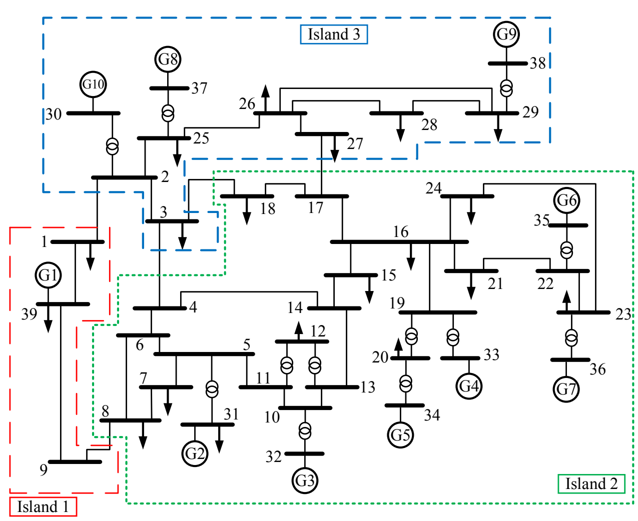

Figure 3. IEEE 39-bus test system

reactive generation capacity. However, the active load in islands 1 and 2 is more than generation capacity which requires proper load shedding in these islands.

TABLE II. IEEE 39-BUS SYSTEM LOAD-GENERATION BALANCE

Island no.	Active Load (P_L)	Active Gen Capacity (P_G)	Reactive Load (Q_L)	Reactive Gen Capacity (Q_G)
1	12.08*	11	2.28	±3
2	35.91	32.52	9.63	±15.1
3	14.55	24.69	1.97	±9.5

*Values are in p.u. base of 100MVA.

C. *IEEE 118-Bus System*

The single-line diagram of IEEE 118-bus system is depicted in Fig. 4. The three coherent generator groups are indicated in table III [24].

By applying the proposed algorithm to the IEEE 118-bus test system, three islands are obtained as depicted with dashed lines in Fig. 4. The suggested cutset includes lines which are shown in Fig. 4 with different color. The total cost of suggested cutsets is 2.55 p.u. As expected, the suggested islanding solution satisfies generator coherency constraints. In addition, the load-generation balance could be preserved without any need of load shedding. As it can be seen in table IV, the active and reactive loads in both islands are less than the active and reactive generation capacity.

TABLE III. IEEE 118-BUS SYSTEM GENERATOR COHERENCY GROUPS

Group 1	Group 2	Group 3
10,12,25,26,31	46,49,54,59,61, 65,66,69,80	87,89,100,103,111

TABLE IV. IEEE 118-BUS SYSTEM LOAD-GENERATION BALANCE

Island no.	Active Load (P_L)	Active Gen Capacity (P_G)	Reactive Load (Q_L)	Reactive Gen Capacity (Q_G)
1	9.63*	14.69	3.45	±23.18
2	27.58	31.47	7.42	±33.69
3	8.50	14.39	3.51	±22.96

*Values are in p.u. base of 100MVA.

D. *Computational Efficency*

Due to enormous size of actual power system, a proper control islanding solution should be computationally efficient while it suggests convenient islanding cutsets. The methods which rely on minimal power imbalance objective function are categorized as NP-hard problems. Since the required computational time to solve such problems is of an exponential order, they cannot be solved in polynomial time

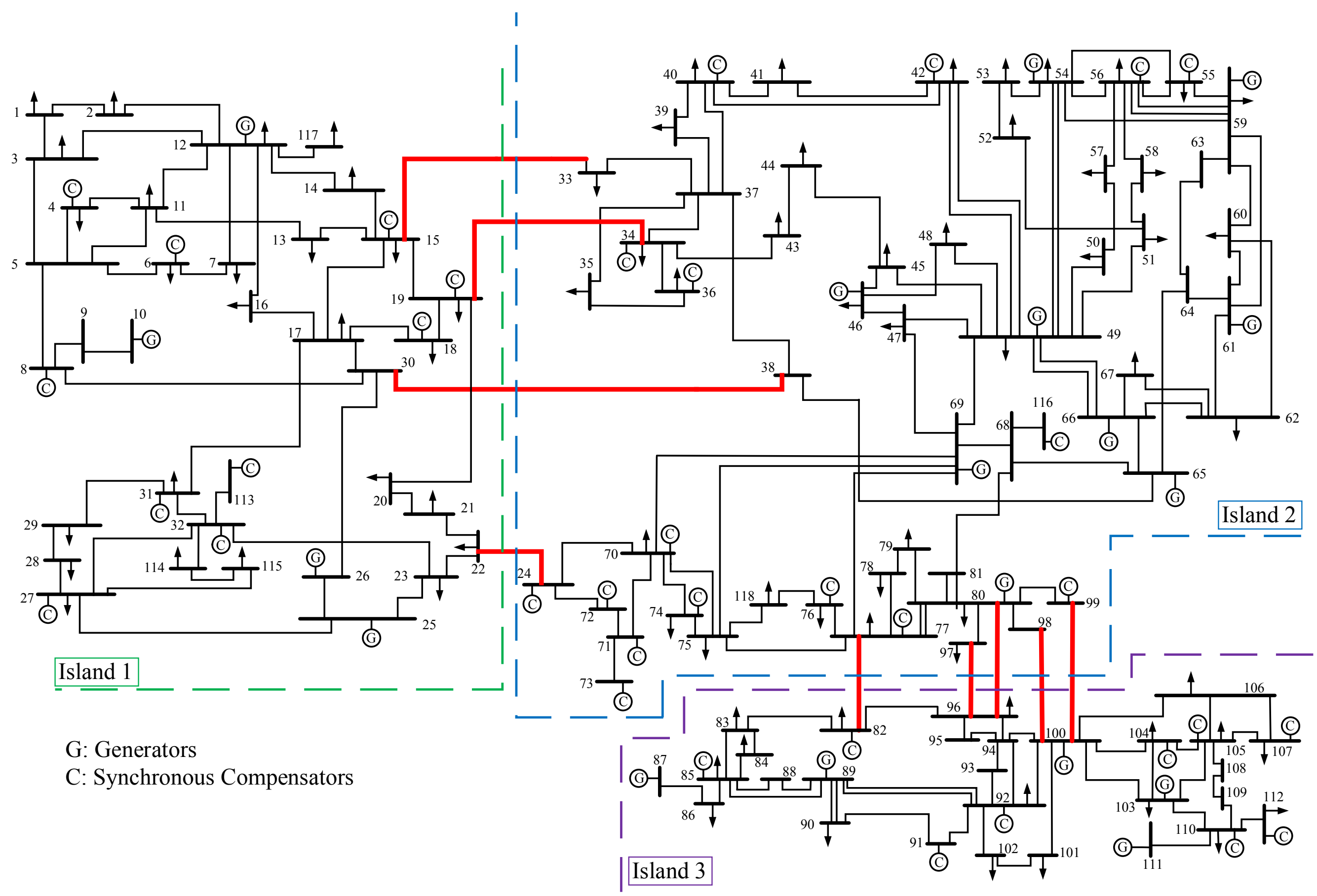

Figure 4. IEEE 118-bus test system

[28]. However, the required time to solve objective function defined based on minimal power-flow disruption such as the one we presented in this paper is in polynomial time order. Hence, this problem can be evaluated as a max-flow-min-cut problem and be solved within polynomial time [28, 29].

Table V summarizes the required time to execute the proposed method using MATLAB R2012b for each of test systems. The simulation is performed using Windows 7, Intel(R) Xeon(R) 3.2GHz CPU, 12GB RAM PC.

TABLE V. PROCEESING TIME REQUIRED BY THE PROPOSED METHOD

Test Systems	IEEE 9-bus	IEEE 39-bus	IEEE 118-bus
Time (s)	<0.0001	0.0032	0.0973

V. CONCLUSION

In this paper, a new constrained spectral k-embedded clustering method is proposed to determine feasible controlled islanding solution for any given power network. The main contribution of this study can be itemized as follows.

- Defining objective function based on minimal power-flow disruption resulted in creation of stable islands; meanwhile it reduced the complexity of islands' re-connection.
- Using spectral k-embedded clustering improves partitioning process by giving more flexibility to adjust clusters.
- Using constraint matrix to include generator coherency information prevents non-coherent generators to fall into same islands.
- Using minimal power-flow disruption, the results indicate that the presented algorithm is time-efficient and can operate as a real-time application to prevent major blackouts in power systems.

REFERENCES

[1] "Final Report on the August 14, 2003 Blackout in the United States and Canada: Causes and Recommendations," U.S.-Canada Power System Outage Task Force, April 5th, 2004.

[2] A. Esmaeilian, T. Popovic, M. Kezunovic, "Transmission Line Relay Mis-operation Detection Based on Time-Synchronized Field Data," *Electric Power Systems Research*, Vol. 125, pp.174-183, 2015.

[3] M. Davoudi, J. Sadeh, E. Kamyab, "Parameter-free fault location for transmission lines based on optimization," *IET Generation, Transmission & Distribution,* vol. 9, no. 11, pp. 1061-1068, Aug. 2015.

[4] P. Dutta, A. Esmaeilian, M. Kezunovic, "Transmission-Line Fault Analysis Using Synchronized Sampling," *IEEE Trans. Power Delivery*, Vol. 29, No. 2, April 2014.

[5] R. Baldick, B. Chowdhury, I. Dobson, D. Zhaoyang, G. Bei, D. Hawkins, et al., "Initial review of methods for cascading failure analysis in electric power transmission systems," in: Proceedings of the IEEE PES General Meeting, Pittsburgh, PA, USA, Jul, 2008.

[6] B. Yang, V. Vittal, G. T. Heydt, "Slow-Coherency-Based Controlled Islanding-A Demonstration of the Approach on the August 14, 2003 Blackout Scenario," *IEEE Transactions on Power Systems,* vol. 21, no. 4, pp. 1840–1847, Nov. 2006.

[7] V. Vittal, W. Kliemann, Y. Ni, D. G. Chapman, A. D. Silk, and D. J. Sobajic, "Determination of generator groupings for an islanding scheme in the Manitoba hydro system using the method of normal forms," *IEEE Transactions on Power Systems*, vol. 13, no. 4, pp. 1345–1351, Nov. 1998.

[8] J. Jung, C. C. Liu, S. L. Tanimoto, and V. Vittal, "Adaptation in load shedding under vulnerable operating conditions," *IEEE Transactions on Power Systems*, vol. 17, no. 4, pp. 1199–1205, Nov. 2002.

[9] H. You, V. Vittal, and Z. Yang, "Self-healing in power systems: an approach using islanding and rate of frequency decline-based load shedding," *IEEE Transactions on Power Systems*, vol. 18, no. 1, pp. 174–181, Feb. 2003.

[10] H. You, V. Vittal, and X. Wang, "Slow coherency-based islanding," *IEEE Transactions on Power Systems*, vol. 19, no. 1, pp. 183–491, Feb. 2004.

[11] K. Sun, D. Zheng, and Q. Lu, "Splitting strategies for islanding operation of large-scale power systems using OBDD-based methods," *IEEE Transactions on Power Systems*, vol. 18, no.2, pp. 912–923, May 2003.

[12] Q. Zhao, K. Sun, D. Zheng, J. Ma, Q. Lu, "A study of system splitting strategies for island operation of power system: a two-phase method based on OBDDs," *IEEE Transactions on Power Systems,* vol. 18, no. 4, pp. 1556–65, Nov. 2003.

[13] H. Li, G. W. Rosenwald, J. Jung, C.-C. Liu, "Strategic Power Infrastructure Defense," *Proceedings of the IEEE*, vol. 93, pp. 918–933, 2005.

[14] A. Peiravi, R. Ildarabadi, "A Fast Algorithm for Intentional Islanding of Power Systems Using the Multilevel Kernel k-Means Approach," Journal of Applied Sciences, vol. 9, pp. 2247–2255, 2009.

[15] L. Liu, W. Liu, D. A. Cartes, I. Y. Chung, "Slow coherency and angle modulated particle swarm optimization based islanding of large-scale power systems," *Advanced Engineering Informatics,* vol. 23, no. 1, pp.45–56, 2009.

[16] J. H. Chow, Time-Scale Modeling of Dynamic Networks with Applications to Power Systems. New York: *Springer*- Verlag, 1982.

[17] M. Davoudi, J. Sadeh, E. Kamyab, "Time domain fault location on transmission lines using genetic algorithm," *11th International Conference on Environment and Electrical Engineering*, pp. 1087-1092, May 2012.

[18] M. A. M. Ariff, B. C. Pal, "Coherency identification in interconnected power system -An independent component analysis approach," *IEEE Transactions on Power Systems*, vol. 28, no. 2, May 2013.

[19] C. Juarez, A. R. Messina, R. Castellanos, G. Espinosa-Perez, "Characterization of multi-Machine system behavior using a hierarchical trajectory cluster analysis," *IEEE Transactions on Power Systems,* vol. 26, no. 3, pp. 972-981, Aug. 2011.

[20] X. Wang, I. Davidson, "Flexible constrained spectral clustering," *16th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*, pp. 563-572, Washington DC, USA, 2010.

[21] X. Wang, B. Qian, I. Davidson, "On constrained spectral clustering and its applications," *Data Mining and Knowledge Discovery*, pp. 1-29, Sep. 2012.

[22] H. Kuhn, A. Tucker, "Nonlinear programming," *ACM SIGMAP Bulletin,* pp.6-18, 1982.

[23] http://www.mathworks.com/help/stats/kmedoids.html

[24] L. Ding, F. M. Gonzalez-Longatt, P. Wall, V. Terzija, "Two-step spectral clustering controlled islanding algorithm," I*EEE Transactions on Power Systems*, vol. 28, no. 2, May 2013.

[25] A. Esmaeilian, M. Kezunovic, "Fault Location Using Wide Area Measurement of Electromechanical Wave Propagation," *Early Access in IEEE Trans. Power Delivery*, Vol. PP, No. 99, pp. 1-9, Dec. 2015.

[26] [Online]. Available: http://www.ee.washington.edu/research/pstca/.

[27] A. Esmaeilian, M. Kezunovic, "Impact of Electromechanical Wave Oscillations Propagation on Protection Schemes," *Electric Power System Research*, 2016. doi:10.1016/j.epsr.2016.01.002

[28] A. Sen, P. Ghosh, V. Vittal, and B. Yang, "A new min-cut problem with application to electric power network partitioning," *Eur. Trans. Elect. Power*, vol. 19, no. 6, pp. 778–797, 2009.

[29] U. V. Luxburg, "A tutorial on spectral clustering," *Statist. Comput.*, vol. 17, no. 4, pp. 395–416, Dec. 2007.