Synchrophasor Reference Algorithm for PMU Calibration System

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***Abstract*—This paper describes a reference algorithm specifically designed for PMU Calibration System. Contrary to existing DFT-based and curve fitting-based methods, which use a single signal model, the proposed algorithm applies an adaptive mechanism, and switches signal models according to specific signal input. Signals are modeled with parameters with apparent and certain physical meaning, fundamentally avoiding error magnification from derivative calculation for frequency and rate of change of frequency estimation. Levenberg-Marquardt algorithm is used for solving the signal parameters. The test results show that the proposed algorithm has a much higher accuracy than the requirements of IEEE standard C37.118.1, and hence can serve as a reference algorithm in a PMU Calibration System.**

***Index Terms*—Levenberg-Marquardt algorithm, PMU calibration system, reference synchrophasor algorithm, synchrophasor estimation, power system measurements.**

I. Introduction

Synchronized phasor measurements and phasor measurement units (PMUs) have grown to enhance wide-area situational awareness, and have gained wide application in modern power system worldwide. An efficient method of estimating phasor using Discrete Fourier Transform (DFT) was introduced in 1983 [1], and has gained prevalence thanks to its high computational efficiency. The prototype of PMU was built in Virginia Tech, and the first PMU device was built by Macrodyne in 1992. So far, there are more than 1000 PMUs commissioned in the US electric power grid, creating a complex wide-area measurement system. [2]

Multiple versions of IEEE standards have been proposed. In 2011, the original IEEE standard was further revised, and separated into two standards: IEEE C37.118.1-2011 [3] for synchrophasor measurement, and IEEE C37.118.2-2011 [4], for synchrophasor data transfer. In the new standard, a dynamic synchrophasor model with broader scope was used. The standard categorized the PMU into P-Class and M-Class, and added the requirements for PMU dynamic tests. The standards were amended by IEEE C37.118a-2014 [5] in 2014. IEEE Synchrophasor Measurement Test Suite Specification [6] was published in 2014 as an unambiguous guidance of PMU testing.

Industry, academia and government are contributing to synchrophasor development and PMU calibration systems together. North America SynchroPhasor Initiative (NASPI) [7] carried work under the auspices of the North America Reliability Corporation (NERC), and has proposed several guides for PMU testing and certification [8]. Funded by the National Institute of Standard and Technology (NIST), DOE, and DOC, SynchroMetrology Laboratory was established at NIST in 2006, the static PMU Calibration System was built in 2008, and the dynamic PMU Calibration System is constructed in 2009 [9]. Besides NIST, many universities and institutes are developing PMU calibration systems and testing laboratories [10]-[13].

A great number of papers have introduced synchrophasor estimation methods in the past three decades. References [14] and [15] proposed interpolated DFT method in frequency domain by estimating the discrepancy between local maxima and DFT bins. This method works only for static multi-frequency signal. Reference [16] analyzed the spectral leakage and averaging effect of DFT, concluding that frequency domain methods cannot perform well if subject to dynamic input. By introducing dynamic phasor model to improve estimation accuracy, references [17] and [18] used Taylor polynomials to fit dynamic signals in time domain, and then calculate synchrophasor using fitting parameters. Time domain methods essentially acknowledge the dynamic nature of synchrophasor. The computational efficiency of this method was improved in [13] by introducing Legendre polynomials. Moreover, Model-based Kalman filter was used for dynamic phasor estimation [19].

The aforementioned methods are capable of performing well under certain test signals, but none of them can keep high accuracy for all test signals, which is required in a PMU calibration system. The algorithm that we propose in this paper, however, takes into account each test signal property,

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and matches the model to the signal parameters for that case, hence can achieve high accuracy for all test scenarios.

The rest of the paper is organized as follows, Section II reviews the structure of PMU calibration system that uses a new PMU algorithm as a reference. Nonlinear regression methods, especially Levenberg-Marquardt algorithm, are introduced in Section III. In Section IV, a new reference synchrophasor estimation method based on nonlinear regression method is presented. Algorithm implementation and test results are presented in Section V.

II. PMU Calibration System Based on Reference PMU

A. *Requirement Specification of a Calibration System*

A PMU Calibration System is a platform for testing and calibration of PMU devices before their commissioning in a power grid. Typically a PMU Calibration System consists of the following subsystems: signal generation, timing reference, PMU measurement receiver, PMU under test, synchrophasor reference source, and result documentation. Structure of a PMU Calibration system is shown in Fig. 1.

Although there are various ways to implement the PMU Calibration System, the basic mechanism of PMU testing is using the reference value to evaluate measurements from PMU under test, and then by comparing reference values and measurements according to the corresponding timestamps.

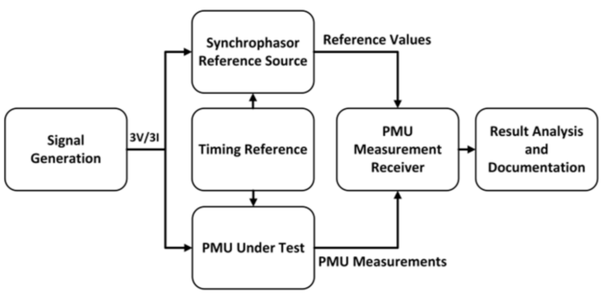

Figure 1. Structure of a typical PMU Calibraiton System

B. *Reference PMU*

According to [7], based on the choice of reference values, three methods can be applied to implement synchrophasor reference source: Direct Measurement, Inference, and Transfer Measurement. In Transfer Measurement method, a devices with higher accuracy, typically with a test uncertainly ratio (TUR) greater than 4, is utilized as the source of reference values. In this paper, Transfer Measurement is adopted where the Reference PMU algorithm serves as the synchrophasor reference. The accuracy of Reference PMU can be achieved by carefully choosing the correct hardware and implementing high precision synchrophasor estimation methods. The inputs and output of a Reference PMU are shown in Fig. 2.

When utilizing a Reference PMU, test scenario input can be used to identify test signal input so that Reference PMU can select the most suitable signal model and algorithm for each input signals to achieve highest estimation accuracy. The proposed algorithm is suitable for either offline or online PMU testing as long as the input signal type is controlled and known in advance.

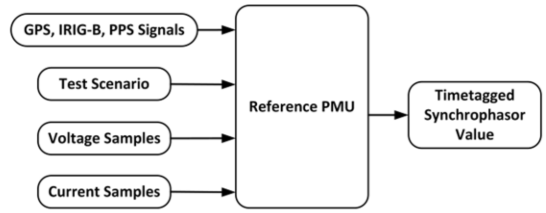

Figure 2. Input and Output of a typical Reference PMU

III. Nonlinear Regression Methods

In 2011, a dynamic synchrophasor model is proposed in IEEE Standard C37.118-2011, as shown in (1).

$$x(t) = \sqrt{2}X_{rms}\,cos(2\pi f_0 t + 2\pi \int \Delta f(t)dt + \varphi_0) \quad (1)$$

where instant phase angle $\varphi(t) = 2\pi \int \Delta f(t)\mathrm{d}t + \varphi_0$ is described as an offset from cosine wave at the nominal frequency. From this definition, phase angle is an accumulation of angular velocity deviation over time.

In our approach, power system signals, which are combination of nonlinear trigonometric functions, can be modeled by (2),

$$\boldsymbol{b} = f(\boldsymbol{t}, \boldsymbol{x}) \quad (2)$$

where $\boldsymbol{b}$ is the known data sample from one data window, $\boldsymbol{t}$ is the unknown time vector corresponding to $\boldsymbol{b}$, and $\boldsymbol{x}$ is the unknown fitting parameter vector. f represents the model of power system signals.

Solving (2) is essentially a nonlinear regression problem. The objective function is given by (3),

$$\text{minimize}_{\boldsymbol{x}}\ \text{SSR}(\boldsymbol{x}) \quad (3)$$

where SSR, or Sum of Squared Residue, is given by (4),

$$\text{SSR}(\boldsymbol{x}) = \left\|\boldsymbol{b} - \boldsymbol{b}_{opt}\right\|_2 = \left\|\boldsymbol{b} - \boldsymbol{b} = f(\boldsymbol{t}, \boldsymbol{x}_{opt})\right\|_2 \quad (4)$$

where the subscript *opt* indicates optimized values to be attained.

A common strategy of solving nonlinear regression problems is to linearize (2) and then solve linear regression problems instead, which is much less complicated. Typical nonlinear regression methods include Gauss-Newton Algorithm (GNA), method of gradient descent (also steepest descent). GNA is not suitable for iteration when the initial condition is far off from the actual value. This drawback can be compensated by method of gradient descent. However, gradient descent method shows a poor converging quality around true value. Levenberg-Marquardt algorithm (LMA) [20] uses a damping factor μ to adjust iteration increment to find a trade-off between GNA and gradient descent method. For a nonlinear equation (2), the increment of LMA is shown in (3).

$$\Delta\boldsymbol{x}^{(i)} = \left[\boldsymbol{J}^{(i)T}\boldsymbol{J}^{(i)} + \mu^{(i)}\text{diag}\left(\boldsymbol{J}^{(i)T}\boldsymbol{J}^{(i)}\right)\right]^{-1}\boldsymbol{J}^{(i)T}\left[\boldsymbol{b} - f(\boldsymbol{t}, \boldsymbol{x}^{(i)})\right] \quad (5)$$

where $\boldsymbol{J} = \partial f / \partial \boldsymbol{x}$ is the Jacobian matrix.

The damping factor μ is chosen to be a relatively large value at the beginning of iteration, where the initial value tends to be away from the real value. The iteration increment is large, which is similar to gradient descent method, for a fast approach of real value; then μ decrease and LMA becomes similar to GNA, in order to get more accurate result.

IV. Synchrophasor Algorithms for Reference PMU

The drawback of the existing curve fitting based, time domain methods is that there is no physical meaning of the fitting parameters. The quantities with physical meaning, namely amplitude, phase angle, frequency, and rate of change of frequency, are derived from the fitting parameters. As a result, the input signal cannot be modeled in a way that the model truly reflects the input signal. Moreover, usually the frequency and rate of change of frequency (ROCOF) are acquired by taking the derivative and second derivative of phase angle, respectively. This process, however, magnifies the error in phase angle estimation.

A general rule of thumb is, the more information about the signal to be estimated can be acquired, the more accurate estimation results can be expected. As stated above and shown in Fig. 2, Test Scenario should be considered as a known parameter to the PMU Calibration System. The algorithm proposed in this paper employs the additional Test Scenario input to identify the test signals, and apply respective signal models for each scenario. Nonlinear least square method is used to calculate fitting parameters in the models.

A. *Procedure of Synchrophasor Estimation using Proposed Algorithm*

According to IEEE standards, a PMU has to be subjected to static and dynamic tests. Details of the test types as well as the corresponding power system scenarios are listed in Table 1. Diagram of how the algorithm works as a reference algorithm is shown in Fig. 3.

TABLE I. Test Scenarios in PMU Tests

Test Types		**Power System Scenarios**
Static Tests	***Signal frequency range***	Frequency deviation under normal condition
	Signal magnitude range	Magnitude levels under normal condition
	Phase angle range	Normal condition featuring a slowly varying angle
	Harmonic distortion	Harmonic infiltration from power electronic devices, etc.
	Out-of-band interference	Testing PMU anti-aliasing effectiveness
Dynamic Tests	***Amplitude modulation***	Amplitude oscillation, low Frequency oscillation
	Phase modulation	Frequency oscillation, subsynchronous resonance
	Frequency ramp	Generator out-of-step
	Input step change	Faults in power grids

As shown in Fig.3, during a PMU test, the proposed reference algorithm uses Test Scenario to identify input signal and select corresponding signal model, which is introduced in the following sections. Then LMA performs nonlinear regression method to estimate model parameters. The signal model is chosen in an optimized way so that the most prior knowledge of input test signal is employed for improved accuracy.

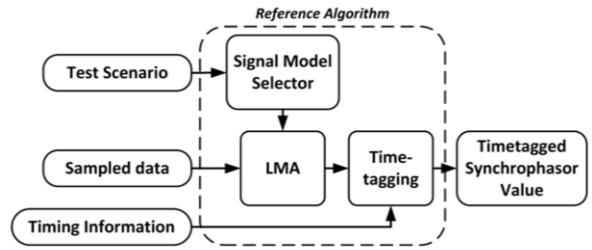

Figure 3. Procedure of synchrophasor estimation with proposed algorithm

B. *Model for Static Signals and Frequency Ramp Signals*

In static and frequency ramp signals, the parameters of particular interest are: root-mean-square value X_{rms}, initial angle φ_0 instant frequency deviation from nominal frequency Δf), and rate of change of frequency (ROCOF) R_f.

$$x(t) = \sqrt{2} X_{rms} \cos\left(2\pi f_0 t + 2\pi \Delta f t + \pi R_f t^2 + \varphi_0\right) \quad (6)$$

where f_0 denotes nominal frequency. The corresponding synchrophasor of signal in (6) is,

$$p(t) = X_{rms} e^{j(2\pi \Delta f t + \pi R_f t^2 + \varphi_0)} \quad (7)$$

C. *Model for Harmonic Distorted and Out-of-Band Signals*

In signal with harmonic distortion model, an additional harmonic term is added to (6). The harmonic signal is modeled as a single frequency sinusoid.

$$x(t) = \sqrt{2} X_{rms} \cos\left(2\pi f_0 t + 2\pi \Delta f t + \pi R_f t^2 + \varphi_0\right) + \sqrt{2} X_{rms,har} \cos\left(2\pi k f_0 t + 2\pi k \Delta f_{har} t + \varphi_{0,har}\right) \quad (8)$$

where k represents the order of harmonic, subscript $*_{har}$ denotes the parameters for harmonic signal.

The corresponding synchrophasor for (8) is the same as (7), since harmonic signal is not taken into consideration in synchrophasor model.

The out-of-band (OOB) test signals can also be considered as a signal harmonic input. IEEE standard requires that the PMU can withstand harmonics from 2^{nd} order to 50^{th} order, and OOB signal from passband to 2^{nd} harmonics. [5]

D. *Model for Modulation Signals*

IEEE standard [5] specifies two types of modulation tests, amplitude modulation and frequency/phase modulation. Modulation parameters are added to (6).

$$x(t) = \sqrt{2} X_{rms} [1 + k_m \cos(2\pi f_m t)] \cos[2\pi f_0 t + 2\pi \Delta f t + k_a \cos(2\pi f_m t - \pi) + \varphi_0] \quad (9)$$

where k_m is the amplitude modulation level, k_a is the frequency modulation level, f_m is modulation frequency.

The corresponding synchrophasor model for (9) is,

$$p(t) = X_{rms}[1 + k_m \cos(2\pi f_m t)] \, e^{j(2\pi\Delta f t + k_a \cos(2\pi f_m t - \pi) + \varphi_0)} \quad (10)$$

Note that frequency deviation Δf is added to take into account the frequency error of signal generator. Frequency of (9) is the first derivative of the phase angle,

$$f(t) = \frac{1}{2\pi}\frac{d\varphi(t)}{dt} = \Delta f - f_m k_a \sin(2\pi f_m t - \pi) \quad (11)$$

ROCOF of (9) is calculated by taking the derivative of (11),

$$R_f = \frac{df(t)}{dt} = -2\pi f_m^2 k_a \cos(2\pi f_m t - \pi) \quad (12)$$

E. Model for Input Step Signals

Input step test signals can be considered as a concatenation of two steady state sinusoids. Therefore the model in (6) is used.

However, input step tests focus on evaluating the response time and delay time of the algorithm, rather than the estimation accuracy. Hence, a Reference PMU should be able to accurately detect and locate the moment of a step change. Since the transition time of an algorithm is usually equal to the length of data used for estimation, shorter data window (preferably shorter than one cycle) should be used. To compensate the loss of available data, higher sampling rate should be utilized.

V. Algorithm Implementation and Testing

A. Reference PMU Setup

The Reference PMU implementation at Texas A&M University is based on National Instruments PXI platform, which is composed of a 2.0GHz dual-core embedded controller, analog acquisition card, and a timing card capable of decoding IRIG-B signal. Timing signal is provided by Symmetricom clock. Software for reference PMU, including users' interface, reference algorithms, and hardware configuration codes, is written in National Instruments LabVIEW. The hardware system is shown in Fig. 4.

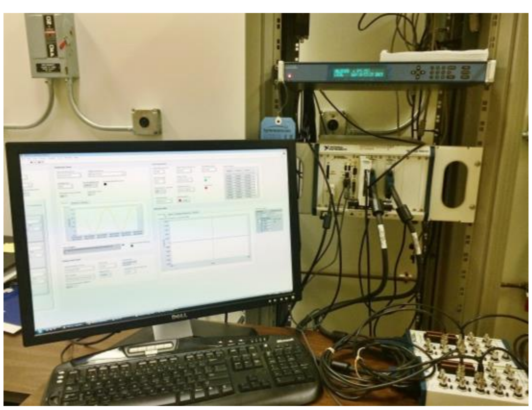

Figure 4. Reference PMU Test System at Texas A&M University

B. Iteration Flowchart

As introduced in Section III and IV, the proposed synchrophasor reference algorithm utilizes different nonlinear signal models for respective input signals. As a result, LMA is needed to acquire the model parameters. LMA requires iteration and initial values input of fitting parameter vector $\boldsymbol{x}$, as in (2), and damping factor μ, as in (5). Flowchart of iteration process is shown in Fig. 5.

C. Test Conditions

While test signals specified in [6] are used, two set of tests are performed to evaluate the accuracy of the proposed reference algorithm. The first set of tests is theoretical simulation tests, where the algorithm is tested under theoretical input in LabVIEW. The second set of tests is implementation test, where white Gaussian noise (WGN) is added to the theoretical signal to simulate the effect of digitization and sampling noise. An effective-number-of-bits (ENOB) of 14-bit is chosen to represent the actual 16-bit ADC resolution, which is equivalent to a signal-to-noise ratio of 70dB WGN [18]. Sampling noise is modeled according to [21].

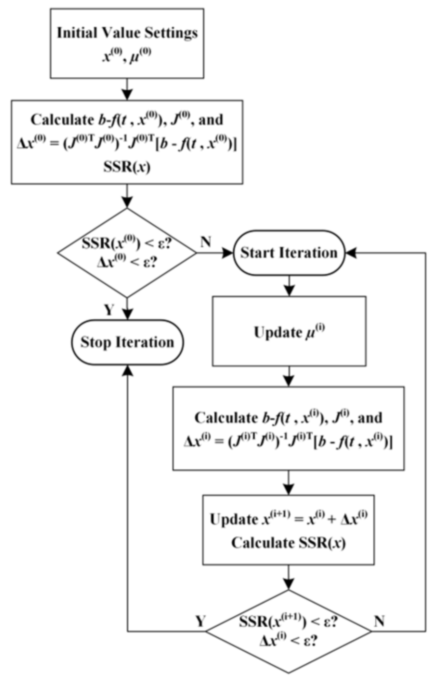

Figure 5. Flowchart of Reference Algorithm

In the theoretical simulation test, the following test scenarios are considered: steady-state tests, harmonic test, out-of-band test, frequency ramp test, modulation test. The signal models are chosen according to Section IV.

For the implementation test, same test scenarios are used as the theoretical simulation tests. However, white Gaussian noise is added to the input signals.

D. Theoretical Simulation Test Results

As is shown in Table II, the proposed reference algorithm presents high accuracy in typical tests specified in IEEE standards. Since frequency and ROCOF are specifically modeled, they can present the same accuracy level as amplitude and angle estimation, which cannot be achieved by traditional algorithms. Note that the algorithm is designed to comply with requirements for M-class PMU, but for

Harmonic and OOB tests, only P-class ROCOF requirements are available so far.

E. Implementation Test Results

As is shown in Table III, the proposed reference algorithm can remain adequate accuracy for PMU Calibration System. Thanks to customized modeling of input signal, frequency and ROCOF estimation exhibits high accuracy. For modulation test, ROCOF is not modeled as unknown parameter, thus it is calculated by taking the derivative of frequency, and this is how estimation error is magnified.

TABLE II. ALGORITHM ACCURACY IN THEORETICAL SIMULATION TESTS

Test Type	TVE/IEEE (%)	Frequency/IEEE (Hz)	ROCOF/IEEE (Hz/s)
Steady-state test	10^{-5}%/1%	10^{-5}/0.005	10^{-5}/0.1
Harmonic test	10^{-5}%/1%	10^{-5}/0.005	2×10^{-5}/0.4 (P)
OOB test	10^{-5}%/1.3%	10^{-5}/0.005	2×10^{-5}/0.4 (P)
Frequency ramp test	5×10^{-4}%/1%	5×10^{-6}/0.01	3×10^{-5}/0.2
Modulation test	5×10^{-5}%/3%	2×10^{-6}/0.06	10^{-3}/2

TABLE III. ALGORITHM ACCURACY IN IMPLEMENTATION TESTS

Test Type	TVE/IEEE (%)	Frequency/IEEE (Hz)	ROCOF/IEEE (Hz/s)
Steady-state test	0.01%/1%	0.0004/0.005	5×10^{-4}/0.1
Harmonic test	0.01%/1%	0.002/0.005	10^{-3}/0.4 (P)
OOB test	0.01%/1%	0.002/0.005	10^{-3}/0.4 (P)
Frequency ramp test	0.04%/1%	0.002/0.01	4×10^{-4}/0.2
Modulation test	0.09%/3%	0.012/0.06	0.2/2

VI. CONCLUSION

In this paper, a reference synchrophasor algorithm for PMU Calibration System is presented. The conclusions are as follows.

- Input test signals are modeled with parameters with specific physical meaning for accurate description of test signals.
- Test scenario is used as an additional known input to identify and switch respective signal models, so that higher estimation accuracy can be achieved.
- Levenberg-Marquardt algorithm is adopted to perform nonlinear parameter estimation. The proposed algorithm is tested against an algorithm provided in the IEEE standard. Test results show high accuracy of the proposed algorithm.
- Differentiation of phase angles, which magnifies angle estimation error, is avoided during frequency and ROCOF estimation. Hence, higher accuracy of frequency and ROCOF estimation is achieved.
- The proposed algorithm requires prior knowledge of test scenario and employs iteration method, which can be used in a PMU Calibration System for online/offline PMU testing with controlled signal input.

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