A Wavelet Method for Power System Frequency and Harmonic Estimation

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***Abstract*—This paper proposes a new method for estimating frequency and harmonic parameters in power system. The method is based on recursive wavelet transform and uses the wavelet transform coefficients of voltage and current signals. The complex magnitudes contain the phasor information of interest needed to estimate the frequency and selective harmonic components. This approach is capable of accurately estimating the frequency and selective harmonic parameter utilizing one cycle of an input signal. It features fast response and achieves high accuracy over a wide range of frequency variation. Effects of the sampling frequency and data window size on the algorithm performance are investigated. The two parameters can be selected to meet desirable applications requirements, such as fast response, high accuracy and low computational burden. Simulation results demonstrate that the proposed method achieves good performance.**

***Index Terms*—Frequency measurement, harmonic estimation, waveform distortion, recursive wavelet transform (RWT)**

I. Introduction

THE increasing use of power electronic devices and non-linear loads causes harmonic pollutions and other high frequency contaminations in power system. The harmonics can cause undesirable effects such as overheating equipment, increasing losses, vibrating of rotating machines, misleading protective relay operation and producing interferences to communication devices [1]. The difficulty of tracking frequency in power networks has also grown more complex due to the harmonic contamination. Compensation techniques and corresponding devices have been developed to improve the power quality [2], [3]. It was learned that quickly and accurately estimating the frequency and harmonic components of an input signal may help better eliminating their effects on the power quality.

Discrete Fourier Transform (DFT) based methods have been extensively used as a spectral analysis tool for frequency and harmonic evaluation due to the low computational requirement [4], [5]. The methods achieve good performance when the signals are stable and the frequency components of interest are constant spanning the data window. However, the inherent limitation such as the spectral leakage, resolution and the implicit data window requirements in DFT approach causes errors when frequency deviates from the nominal value [6].

To improve the performance of DFT based approaches, some adaptive methods based on feedback loop by tuning the sampling interval [7], adjusting data window length [8], changing the nominal frequency used in DFT iteratively [6], and correcting the gains of orthogonal filters recursively [9] are proposed. On the basis of stationary signal model, some non-linear curve fitting techniques, including extended Kalman filter [10] and recursive Least Squares algorithm [11], are adopted to estimate the power network frequency. The accuracy is only reached in a narrow range around nominal frequency due to the truncation of Taylor series expansions of nonlinear terms.

Wavelet transform has been used to detect the power quality disturbances as the high frequencies associated with disturbances can be identified and localized in both time and frequency domain using wavelet transform coefficients [12], [13]. The method performance varies based on the characteristics of different wavelets being utilized.

The artificial intelligence techniques, such as genetic algorithm [14], [15], neural networks [16], [17], and fuzzy regression [18] have been used to achieve precise frequency estimation over a wide range and under harmonic contamination. Although better performance can be achieved by these optimization techniques, the implementation algorithm is more complex and computationally more intensive.

Recursive wavelet approach was introduced in protective relaying for a long time [19], [20]. The improved model with single direction recursive equations is more suitable for the application to real-time signal processing [21]. The band energy of any center frequency can be extracted through recursive wavelet transform (RWT) with moderately low computation load.

The paper presents a novel approach for frequency and harmonic estimation based on RWT. The frequency and harmonics of interest can be accurately estimated through iteration. The approach can respond in once cycle of a nominal frequency signal. The convergence characteristics associated with the sampling frequency and data window size are studied. It may be observed that the higher the sampling rate, the shorter the data window size the computation needs, and vice versa. Analysis indicates that the computational burden is fairly low due to the recursive formulae. Performance test results including steady-state and dynamic cases demonstrate the advantages of the proposed method.

The paper is organized as follows. Section II introduces characteristics of the recursive wavelet transform both in time

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and frequency domain, and describes the frequency and harmonic estimation algorithm. The analysis of convergence and computation burden is presented in Section III. Section IV presents the details of performance evaluation. Conclusions are outlined at end.

II. Frequency and Harmonic Estimation

In this section, the basics of recursive wavelet transform are briefly introduced. Then the derivation of the frequency and harmonic estimation algorithm are described in detail.

A. Recursive Wavelet Transform

A mother wavelet function that satisfies the admissibility condition is expressed as follows [21]:

$$\psi(t) = \left(\frac{\sigma t}{2} + \frac{\sigma^2 t^2}{2} + \frac{\sigma^3 t^3}{3}\right) e^{(\sigma + j\omega_0)t} u(-t)$$

A set of wavelet functions can be derived from $\psi(t)$ by dilating and shifting the mother wavelet, as given below:

$$\psi_{a,b}(t) = a^{-1/2} \cdot \psi\left(\frac{t-b}{a}\right), a > 0$$

where a and b are scaling (dilation) factor and time shifting (translation) factor, respectively.

Its frequency domain expression obtained by Fourier transform is given in following expression:

$$\Psi(\omega) = \frac{\sigma(\omega_0 - \omega)^2 - 3\sigma^3}{2 \cdot [\sigma + j(\omega_0 - \omega)]^4}$$

Setting $\sigma = 2\pi / \sqrt{3}$, $\omega_0 = 2\pi$ makes the wavelet function $\psi(t)$ admissible, i.e. $\Psi(\omega)|_{\omega=0} = 0$.

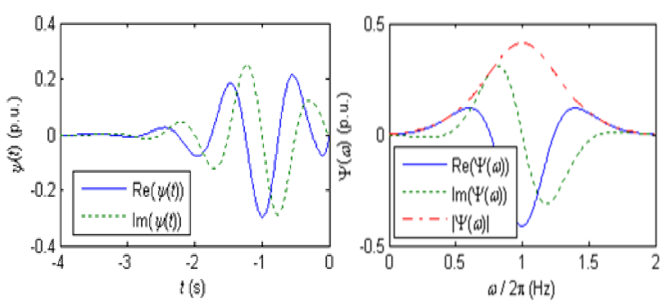

Fig. 1 Time $\psi(t)$ and frequency $\Psi(\omega)$ domain waveforms

The wavelet is a complex function whose time and frequency domain expressions contain real and imaginary parts. Fig. 1 gives time and frequency domain waveforms of $\psi(t)$ and $\Psi(\omega)$, respectively. The wavelet in frequency domain features a band-pass filter with the frequency-domain center ω^* and band radius $\Delta\omega$ of 2π rad and 1.38 rad. One of advantages of the wavelet transform is that the quality factor Q, defined as the ratio of frequency center ω^* and bandwidth $2\Delta\omega$, stays constant as the observation scale varies. For $\Psi(\omega)$, $Q = \omega^*/2\Delta\omega = 2.27$. The complex wavelet exhibits good time-frequency localization characteristics. Its time-frequency window area S defined as a product of time window width $2\Delta t$ and frequency band width $2\Delta\omega$ is 2.23 rad·s.

According to the definition of wavelet transform, the coefficient in scale a for a given signal $x(t)$ can be expressed as:

$$W_{x(t)}(a,b) = a^{-1/2} \int_{-\infty}^{+\infty} x(t) \cdot \psi\left(\frac{t-b}{a}\right)^* dt, t > 0$$

Through mathematical derivations, we obtain the recursive expression for discretely computing the wavelet transform coefficients:

$$\begin{aligned} & W_{x(n\Delta T)}(f, k\Delta T) = \Delta T \sqrt{f} [\lambda_1 x((k-1)\Delta T) \\ & + \lambda_2 x((k-2)\Delta T) + \lambda_3 x((k-3)\Delta T)] \\ & - \beta_1 W_{x(n\Delta T)}(f, (k-1)\Delta T) - \beta_2 W_{x(n\Delta T)}(f, (k-2)\Delta T) \\ & - \beta_3 W_{x(n\Delta T)}(f, (k-3)\Delta T) - \beta_4 W_{x(n\Delta T)}(f, (k-4)\Delta T) \end{aligned} \quad (1)$$

where ΔT is the sampling period, n and k are integers. Other constant parameters are given in the Appendix.

In recursive formula (1), f represents the observing center frequency which is reciprocal to the scale factor a. Applying $f = f_0$ to (1) the frequency band energy centered in f_0 can be extracted. The wavelet transform coefficients can be calculated recursively using the historical and current data. Comparing with the WT in [12], [13] and the RWT in [19], [20], this formula requires less computation load.

B. Frequency and Harmonic Estimation Algorithm

The recursive wavelet features a complex wavelet whose wavelet transform coefficients (real part and imaginary part) contain both phase and magnitude information of the input signal, based on which the algorithm for estimating the power system frequency and harmonic parameters is derived as follows. Let us consider a discrete input signal that contains Mth order harmonics:

$$x(n) = \sum_{m=1}^{M} A_m \cos(2\pi \cdot f_m \cdot n\Delta T + \varphi_m), n = 0, 1, 2, \cdots \quad (2)$$

where f_m, A_m and φ_m represent the frequency, amplitude and phase angle of mth order harmonic, respectively. Denote the absolute phase angle of the mth order harmonic at sample n as $\theta_m(n) = 2\pi \cdot f_m n\Delta T + \varphi_m$. For simplicity, the sampling period ΔT is neglected when expressing variables for the rest of the paper.

Applying RWT in scale a using (1) the input signal $x(n)$ can be represented in time-frequency domain. As derived in Appendix we have following expression:

$$\begin{aligned} W_{x(n)}(a,k) &= \sum_{m=1}^{M} u_m^c(a, f_m, k) \cdot x_m^c(k) \\ &+ \sum_{m=1}^{M} u_m^s(a, f_m, k) \cdot x_m^s(k), k = 0, 1, 2, \cdots \end{aligned} \quad (3)$$

From equation (3) one can see that the wavelet transform coefficient $W_{x(n)}$ contains information of input signal in both cosine form and sine form, denoted as x_m^c and x_m^s in equation (4a) and (4b) respectively, multiplied by weighting factors, denoted as u_m^c and u_m^s in equation (5a) and (5b) respectively.

Let $\widetilde{f}_m$ represent the initial estimate of frequency variable, and let us approximate equation (5a) with the first order Taylor expansion. That results in:

$$u_m^c(a, f_m, k) = u_m^c(a, \widetilde{f}_m, k) + \left.\frac{du_m^c}{df_m}\right|_{\widetilde{f}_m} \cdot \Delta f_m$$

$$= u_m^c(a, \widetilde{f}_m, k) + u_m^{c1}(a, \widetilde{f}_m, k) \cdot \Delta f_m$$

$$u_m^{c1}(a, \widetilde{f}_m, k) = -2\pi\sqrt{a}\Delta T^2 \sum_{l=k/a}^{0} l \sin(2\pi \widetilde{f}_m a l \Delta T) \cdot Q \qquad (6a)$$

For simplicity, denote $u_m^c(a, \widetilde{f}_m, k)$ and $u_m^{c1}(a, \widetilde{f}_m, k)$ as $\widetilde{u}_m^c$ and $\widetilde{u}_m^{c1}$ respectively. Then above equation can be rewritten as follows:

$$u_m^c(a, f_m, k) = \widetilde{u}_m^c + \widetilde{u}_m^{c1} \cdot \Delta f_m$$

Following the same procedures we can rewrite equation (5b) as follows:

$$u_m^s(a, f_m, k) = \widetilde{u}_m^s + \widetilde{u}_m^{s1} \cdot \Delta f_m$$

$$\widetilde{u}_m^{s1} = 2\pi\sqrt{a}\Delta T^2 \sum_{l=k/a}^{0} l \cos(2\pi \widetilde{f}_m a l \Delta T) \cdot Q \qquad (6b)$$

Then the equation (3) can be expressed as follows:

$$W_{x(n)}(a,k) = \sum_{m=1}^{M} [\widetilde{u}_m^c \cdot \widetilde{x}_m^c(k) + \widetilde{u}_m^{c1} \cdot \widetilde{x}_m^{c1}(k)]$$

$$+ \sum_{m=1}^{M} [\widetilde{u}_m^s \cdot \widetilde{x}_m^s(k) + \widetilde{u}_m^{s1} \cdot \widetilde{x}_m^{s1}(k)] \qquad (7a)$$

where $\widetilde{x}_m^{c1} = \widetilde{x}_m^c \cdot \Delta f_m$ and $\widetilde{x}_m^{s1} = \widetilde{x}_m^s \cdot \Delta f_m$

After applying RWT to $x(n)$ in a series of scales a_1, a_2, …, a_{4M} we obtain a series of coefficients w_1, w_2, …, w_{4M} that can be expressed in (7a). Rewrite those equations in matrix form:

$$\begin{bmatrix} w_{a_1} \\ w_{a_2} \\ w_{a_3} \\ w_{a_4} \\ \vdots \\ w_{a_{4M}} \end{bmatrix} = \begin{bmatrix} \widetilde{u}_{1,a_1}^c & \widetilde{u}_{1,a_1}^{c1} & \widetilde{u}_{1,a_1}^s & \widetilde{u}_{1,a_1}^{s1} & \cdots & \widetilde{u}_{M,a_1}^{s1} \\ \widetilde{u}_{1,a_2}^c & \widetilde{u}_{1,a_2}^{c1} & \widetilde{u}_{1,a_2}^s & \widetilde{u}_{1,a_2}^{s1} & \cdots & \widetilde{u}_{M,a_2}^{s1} \\ \widetilde{u}_{1,a_3}^c & \widetilde{u}_{1,a_3}^{c1} & \widetilde{u}_{1,a_3}^s & \widetilde{u}_{1,a_3}^{s1} & \cdots & \widetilde{u}_{M,a_3}^{s1} \\ \widetilde{u}_{1,a_4}^c & \widetilde{u}_{1,a_4}^{c1} & \widetilde{u}_{1,a_4}^s & \widetilde{u}_{1,a_4}^{s1} & \cdots & \widetilde{u}_{M,a_4}^{s1} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \widetilde{u}_{1,a_{4M}}^c & \widetilde{u}_{1,a_{4M}}^{c1} & \widetilde{u}_{1,a_{4M}}^s & \widetilde{u}_{1,a_{4M}}^{s1} & \cdots & \widetilde{u}_{M,a_{4M}}^{s1} \end{bmatrix} \cdot \begin{bmatrix} \widetilde{x}_1^c \\ \widetilde{x}_1^{c1} \\ \widetilde{x}_1^s \\ \widetilde{x}_1^{s1} \\ \vdots \\ \widetilde{x}_M^{s1} \end{bmatrix}$$

At sample k the matrix can be represented in vector form:

$$W(k) = \widetilde{U}(k) \cdot \widetilde{X}(k) \qquad (7b)$$

In (6b) the wavelet coefficient $W(k)$ can be calculated using recursive equation (1). For weighting factor $\widetilde{U}(k)$, it can be calculated with estimated frequency $\widetilde{f}_m$ using equations (5a-b) and (6a-b). Solving equation (7b) we obtain vector variable $\widetilde{X}(k)$. Then we can derive the following formula for Δf_m:

$$\Delta f_m = \frac{\widetilde{x}_m^c(k) \cdot \widetilde{x}_m^{c1}(k) + \widetilde{x}_m^s(k) \cdot \widetilde{x}_m^{s1}(k)}{(\widetilde{x}_m^c(k))^2 + (\widetilde{x}_m^s(k))^2} \qquad (8a)$$

After that, we estimate the frequency adjustment, update frequency with $\widetilde{f}_m + \Delta f_m$ and iterate the above approximation procedures until either the frequency change reaches the cutoff value, for example ε = 0.01 Hz, or a maximum number of iterations denoted as L is performed. As a result, the actual frequency can be estimated at the last iteration. Then, the amplitude A_m and phase angle φ_m can be estimated by the following equations:

$$A_m = \sqrt{\widetilde{x}_m^c(k) + \widetilde{x}_m^s(k)} \qquad (8b)$$

$$\varphi_m = \theta_m(k) - 2\pi \cdot \widetilde{f}_m \cdot k\Delta T \qquad (8c)$$

where $\theta_m(k) = \tan^{-1} \widetilde{x}_m^s(k) / \widetilde{x}_m^c(k)$

C. Implementation Procedure

The following procedure summarizes the implementation flow of the frequency and harmonic estimation algorithm:

1) Calculate wavelet transform coefficients $W(k)$ of input signal using (1) in pre-selected scale factors a_1, a_2, …, a_{4M}.

2) Initialize frequency estimates with $\widetilde{f}_m = m \cdot f_0$ and start iteration.

3) Compute weighting factors $\widetilde{U}(k)$ using (5a-b) and (6a-b).

4) Estimate $\widetilde{X}(k)$ by solving (7b). Then Δf_m can be calculated using (8a) and update frequency with $\widetilde{f}_m + \Delta f_m$.

5) Repeat steps c and d until either $|\Delta f_m| < \epsilon$ or a maximum number of iterations is reached.

6) Estimate the amplitude and phase angle parameters of selected harmonics using (8b) and (8c).

Generally we choose m multiples of nominal frequency (i.e. f_0 = 60 Hz, m represents the order of harmonics) as initial estimate to start iterations. To achieve high accuracy, scale factors [a_1, a_2… a_{4M}] are required to cover all the frequency components of the signal being analyzed. Therefore, we select $a_m = 1 / (f_0 + (4m-1)*f_0/ 4)$. Extensive simulations show that proposed algorithm can converge to the real value within three iterations. It should be noted that if only some harmonic components are of interest, i.e. only f_1, f_3 and f_5 are taken into iteration loop, the dimension of scale factors and weighting matrix will be reduced to eight. Apparently if input signal only contains the fundamental frequency component the solved variables x_m^c and x_m^s ($2 \le m \le M$) will be some numbers close to zero, and then the parameters of those harmonics are meaningless.

III. Analysis of Convergence Characteristic

The sampling frequency and window span may affect the convergence characteristic because of two factors. One is that these formulae are derived based on the assumption that the error resulting from the discrete computation is negligible. Another is the error introduced by the inherent settling process in recursive equations. To analyze the convergence characteristic, we define the window size l_s as the cycle of the nominal frequency, which is independent of the signal sampling frequency f_s defined as N times of nominal frequency f_0 in Hz. Apparently l_s and f_s determine the number of samples N_s within a data window, i.e. $N_s = l_s \cdot f_s / f_0 = l_s \cdot N$.

Total vector error (TVE) is used to measure the accuracy of selective harmonic component [22].

The signal model in (2) is used for the algorithm convergence analysis. In (2) we let f_1 = 60 Hz and M = 5, that is the fundamental frequency component contained in signal is 60 Hz and the frequency of harmonic noise is up to 300 Hz. Analysis results are given in Fig. 2, in which dot represents the convergence while "x" denotes divergence. Results indicate that the algorithm converges in one cycle if the sampling rate is 14 samples per cycle or higher. The window size can be shortened to a half cycle when the sampling frequency is 26 samples per cycle or higher. In practice various noise levels introduced by instrumentation and other interference sources, and the non-stationary frame of input signals may affect the algorithm convergence. Thus we select one cycle as the size of algorithm window. Simulation performed in the following section show that for many cases the estimation method can converge in three iterations.

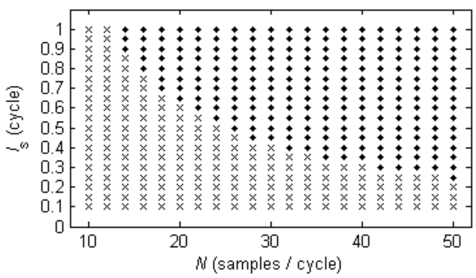

Fig. 2 Convergence analysis results

For implementation, let us only consider the computational burden of the proposed algorithm. If we use 1.2 kHz sampling frequency and one cycle data window (the case performed in convergence analysis) for estimating five harmonic components it approximately requires 6672 multiplications and 6444 summations, in which 124*(2M+2) = 1488 multiplications and 105*(2M+2) = 1260 summations for computing RWT coefficients $W_{2M+2}(12)$ (where M=5), and 3*$(2M+2)^3$ =5184 multiplications and summations for matrix inverse computation when three iterations are performed. Weighting matrix $U(12)$ with various scales and frequencies can be calculated and stored in advance and can be accessed very fast using a table look-up method. Some mathematical techniques such as Chelosky method can be adopted to simplify the matrix computation. Then, the computation burden can be noticeably reduced to 124*(2M+2) + 3*$(2M+2)^2$ = 1920 multiplications and 105*(2M+2) +3*$(2M+2)^2$ = 1692 summations. Based on the analysis one can see that the total computational burden is fairly low. The proposed method, hence, can be used for on-line frequency and harmonic analysis applications.

IV. Performance Evaluation

In this section, performance of the proposed estimation algorithm is fully evaluated with static and dynamic signals. A stationary signal model containing harmonics and noise is used and the performance is verified in a wide range of frequency deviation. Frequency ramps are considered as a dynamic condition to evaluate the capability of the algorithm to follow the input signals. All tests are performed with the sampling rate N = 100 samples per cycle, i.e. f_s = 6 kHz, and data window size l_s = 1 cycle.

The signal model containing (1^{st}, 2^{nd} and 3^{th}) harmonics and 1% white noise is given as follows, where $e(n)$ represents the zero-mean Gaussian noise. Let A = 1 p.u., φ = 5°.The fundamental frequency f_1 varies over a wide range from 55 Hz to 65 Hz in 0.2 Hz steps. Fundamental frequency error and total vector errors of harmonic components are estimated as shown in Fig. 3 and Fig. 4 respectively. Results show a very good accuracy in the presence of harmonics and noise.

$$x(n) = \sum_{m=1}^{3} \frac{A}{m} \cos(2\pi \cdot m \cdot f \cdot n\Delta T + m \cdot \varphi\ ) + e(n)$$

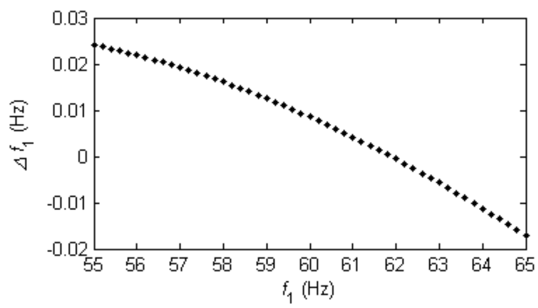

Fig. 3 Fundamental frequency error

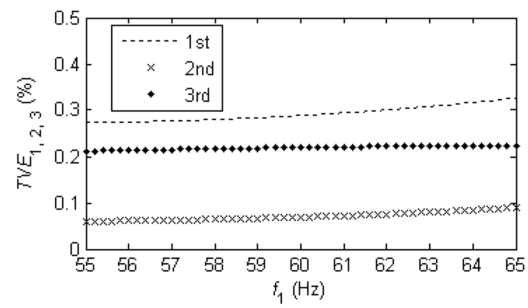

Fig. 4 Total vector error of harmonic components

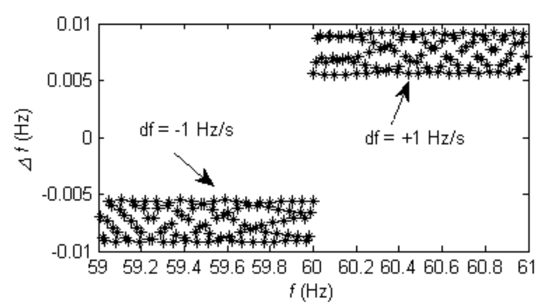

Fig. 5 Frequency ramp test results

The following synthesized sinusoidal signal with a frequency ramp is used to perform the frequency ramp tests. 1% white noise is added as well.

$$x(n) = A \cdot \cos(2\pi \cdot f \cdot n\Delta T + \pi \cdot df \cdot (n\Delta T)^2 + \varphi\ ) + e(n)$$

df is the frequency ramp rate. Both positive ramp +1 Hz / sec and negative ramp -1 Hz/ sec are tested. For positive ramp the frequency of the input signal changes from 60 Hz to 61 Hz in one second while for negative ramp it changes from 60 Hz to 59 Hz in one second. Results in Fig. 5 show that the outputs follow the inputs very accurately.

V. Conclusions

This paper proposes a new method allowing on-line analyzing of power system harmonic components while accurately tracking frequency based on recursive wavelet transform. The algorithm features rapid response and accurate result over a wide range of frequency variation. It uses one cycle of input signals for outputting frequency and harmonic parameters for a signal contaminated with harmonics. Analysis of the algorithm convergence characteristics indicates that the higher the sampling rate, the shorter the computational data window and faster the rate the method outputs results, and vice versa. In practice one cycle window span is used to eliminate the impact of instrumentation noise on the performance. Computational burden analysis indicates that the computation requirement is fairly low. The performance of the proposed algorithm is evaluated using both static and dynamic signal models. Results demonstrate the advantages of the proposed method.

VI. Appendix

The constant parameters in (1) are given as:

$$\lambda_1 = \alpha \cdot [-\sigma f \Delta T/2 + (\sigma f \Delta T)^2/2 - (\sigma f \Delta T)^3/3]$$

$$\lambda_2 = \alpha^2 \cdot [\sigma f \Delta T - 4 \cdot (\sigma f \Delta T)^3/3]$$

$$\lambda_3 = \alpha^3 \cdot [-\sigma f \Delta T/2 - (\sigma f \Delta T)^2/2 - (\sigma f \Delta T)^3/3]$$

$$\beta_1 = -4\alpha,\ \beta_2 = 6\alpha^2,\ \beta_3 = -4\alpha^3,\ \beta_4 = \alpha^4$$

The RWT coefficient of a given signal $x(n)$ is expressed as:

$$W_{x(n)}(a,k) = \frac{\Delta T}{\sqrt{a}} \cdot \sum_{n=0}^{k} x(n) \cdot \psi^*(\frac{n-k}{a}\Delta T),\ k = 0,1,2,\cdots$$
$$= \frac{\Delta T}{\sqrt{a}} \cdot \sum_{n=0}^{k}\sum_{m=1}^{M} A_m \cos(2\pi f_m \cdot n\Delta T + \varphi_m) \cdot [\frac{\sigma}{2}(\frac{n-k}{a})\Delta T$$
$$+ \frac{\sigma^2}{2}(\frac{n-k}{a})^2 \cdot \Delta T^2 + \frac{\sigma^3}{3}(\frac{n-k}{a})^3 \cdot \Delta T^3] \cdot e^{(\sigma - j\omega_0)\cdot(\frac{n-k}{a})\Delta T}$$

Denote $n = l \cdot a + k,\ l \in [-k/a, 0]$, we have,

$$W_{x(n)}(a,k) = \frac{\Delta T}{\sqrt{a}} \cdot \sum_{l=k/a}^{0}\sum_{m=1}^{M} A_m \cos(2\pi f_m k\Delta T + \varphi_m$$
$$+ 2\pi f_m a \cdot l\Delta T) \cdot [\frac{\sigma}{2}(l\Delta T) + \frac{\sigma^2}{2}(l\Delta T)^2 + \frac{\sigma^3}{3}(l\Delta T)^3] \cdot e^{(\sigma - j\omega_0)\cdot l\Delta T}$$

Expand the cosine part and rearrange the equation we obtain,

$$W_{x(n)}(a,k) = \sum_{m=1}^{M} u_m^c(a, f_m, k) \cdot x_m^c(k) + \sum_{m=1}^{M} u_m^s(a, f_m, k) \cdot x_m^s(k)$$

where

$$x_m^c(k) = \sum_{m=1}^{M} A_m \cos(2\pi f_m k\Delta T + \varphi_m) \tag{4a}$$

$$x_m^s(k) = \sum_{m=1}^{M} A_m \sin(2\pi f_m k\Delta T + \varphi_m) \tag{4b}$$

$$u_m^c(a, f_m, k) = \frac{\Delta T}{\sqrt{a}} \cdot \sum_{l=k/a}^{0} \cos(2\pi f_m a \cdot l\Delta T) \cdot Q \tag{5a}$$

$$u_m^s(a, f_m, k) = \frac{\Delta T}{\sqrt{a}} \cdot \sum_{l=k/a}^{0} \sin(2\pi f_m a \cdot l\Delta T) \cdot Q \tag{5b}$$

$$Q = [\frac{\sigma}{2}(l\Delta T) + \frac{\sigma^2}{2}(l\Delta T)^2 + \frac{\sigma^3}{3}(l\Delta T)^3] \cdot e^{(\sigma - j\omega_0)\cdot l\Delta T}$$

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VIII. Biographies

Jinfeng Ren (S’07) received his B.S. degree from Xi’an Jiaotong University, Xi’an, China, in electrical engineering in 2004, and continued studies for Ph.D degree till 2006. He has been with Texas A&M University pursuing his Ph.D degree since Jan. 2007. His research interests are digital simulators and automated simulation methods for protective relay and phasor measurement unit testing and power system control and protection applications.

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