Sensitivity Analysis on the Probabilistic Maintenance Model of Circuit Breaker

Satish Natti, Mladen Kezunovic, *Fellow, IEEE*, and Chanan Singh, *Fellow, IEEE*

***Abstract*--Catastrophic failures of circuit breakers result in high cost associated with loss of load and component replacement. The probability of such failures can be reduced using maintenance procedures. Probabilistic models are needed to estimate failure rate, perform cost benefit analysis and arrive at an optimal maintenance strategy that balances the cost of maintenance with reduction in cost of failures. A probabilistic maintenance model, introduced earlier for circuit breakers, is implemented in this paper. The model parameters are mean time in each stage, inspection rate of each stage, and probabilities of transition from one stage to others. Sensitivity analysis of model parameters is done to establish cost-effective maintenance process. The analysis covers mean time to first failure, probability of failure, maintenance cost, inspection cost, and failure cost. Simulation results are presented.**

***Index Terms*—Circuit Breakers, transformers, inspection, maintenance, probabilistic model, reliability**

I. Introduction

QUANTIFYING the effect of maintenance on reliability is a challenging task in power systems. Reliability analyses and risk analyses often demand the effect of maintenance, especially for devices like power transformers and circuit breakers. Hence there is a need to develop models which relate the maintenance actions to failure rate of the device. Probabilistic models can give more insight of interplay between condition monitoring, inspection and maintenance actions. References [1], [2] introduced probabilistic models to quantify the effect of maintenance on reliability. Based on this general approach along with the concept of device-of-stages [3], power transformer and circuit breaker maintenance models are proposed in [4], [5] respectively. Further, the sensitivity analysis of the transformer model is done in [6].

In this paper, a sensitivity analysis of circuit breaker maintenance model introduced in [5] is performed. The analysis mainly covers the interplay between inspection rate of each stage and the probability of failure. Also, the dependence of various costs (failure cost, maintenance cost, total cost etc.) on inspection rate at each stage is presented. An equivalent model for mathematical analysis is introduced. The simulation results are corroborated with the equations of the equivalent model using steady state probability calculations [3].

The paper is organized as follows. Section II briefly describes the proposed circuit breaker maintenance model. Model parameters are described in section III. Section IV presents the simulation results of sensitivity analysis. Mathematical model analysis using steady state probability calculations is presented in section V. Finally, conclusions are included in section VI.

II. Circuit Breaker Maintenance Model

A probabilistic model, based on the concept of representing the deterioration process by various stages is shown in Fig 1. The model is based on the general probabilistic model of the effect of maintenance on reliability [1]-[2]. Three deterioration stages i.e., the initial stage (D1), minor (D2) and, major (D3), followed by a failure stage are considered.

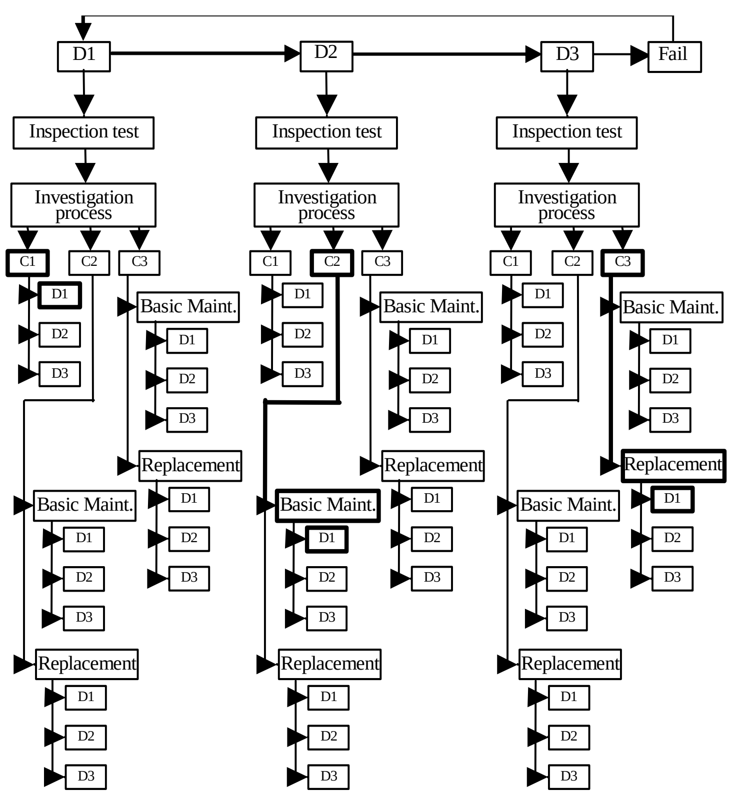

Fig 1: Circuit breaker maintenance model

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Inspection test is implemented at each stage and the collected data is investigated to determine the condition of the breaker. In this model, three different levels of breaker condition are defined: C1- satisfactory and no maintenance is needed, C2- indication of abnormality or caution stage, needs further investigation or related maintenance and C3- Failure stage or poor condition, needs replacement. Further, the maintenance process is divided into three levels; (1) Do nothing, (2) Basic maintenance, and (3) Replacement. Once the suggested maintenance action is taken, the subsequent condition of the breaker is determined.

The model takes results from various inspection and maintenance tasks and the frequency of performing the tasks as inputs and gives the failure rates as output. Reference [5] summarizes inspection and maintenance tasks that are considered in the model. This model can help in obtaining optimum maintenance intervals such that both the component availability and the total cost are balanced.

III. Model Parameters

Table I shows the list and definition of parameters that are used in the circuit breaker maintenance model. The probabilities in model parameter 3 can be treated as equivalent transition rates from one stage to others. The equivalent model is introduced to clarify this point later. Parameters 1 and 3 can be approximated from the historical data of a physical circuit breaker condition [1]. Whereas, parameter 2, which is the inspection rate of each stage can be varied to achieve high reliability with minimum cost. Therefore, this parameter is of the most concern in the analysis. Following section presents the simulation results from MATLAB. Model parameters that are used in the simulation are listed in appendix.

IV. Sensitivity Analysis of Inspection Rate

As discussed in the previous section, the parameter of interest is the inspection rate in each stage.

Let,

i1 = inspection rate of stage 1 (per year)
i2 = inspection rate of stage 2 (per year)
i3 = inspection rate of stage 3 (per year)

From the Fig.1, it can be seen that the inspections which lead back to D1 will not reduce the failure probability; whereas those inspections that lead to D2 and D3 result in degradation. Thus with the assumption that, D1 is exponential distribution, the effect of inspection always results in degradation. It is possible to relax the assumption of exponential distribution by representing the D1 by three sub-stages. The reasons are discussed in detail in [5].

A. Sensitivity analysis of inspection rate on failure probability

Figs. 2-4 show the effect of increasing the inspection rate on probability of failure. Following observation can be made from the simulation results.

1. In Fig 2, for the small values of i1, the failure probability decreases. However, as the i1 increases beyond a number, which can be called as too much inspection, the failure probability increases.

TABLE I
List of Model Parameters and Definitions

Model parameters	Definition
1. Mean time in each stage	It is defined as mean time the device spends in each stage. The inverse of the mean time is the transition rate of the corresponding stage in the deterioration process.
2. Inspection rate of each stage	It is defined as the rate at which the inspection is done. The inspection may be followed by the maintenance.
3. Probabilities of transition from one state to others	These parameters are the probabilities of transition from one state to others. These probabilities include: • The breaker conditions after the inspection process • Transfer from any breaker condition to a given stage • Basic maintenance or replacement • Transferring to each stage after the maintenance

2. Fig 3 and 4 show that the probability of failure decreases with increasing inspection rates, i2 and i3 respectively.

In summary, the simulation results suggest that inspection rate of D1 helps in decreasing the probability of failure; however too much inspection results in increasing failure probability. In this model, the maintenance in stage 1 can result in the system transition to stage 3. Therefore, it is likely that too much maintenance can result in higher failure probability due to problems introduced by maintenance.

B. Sensitivity analysis of inspection rate on all associated costs

Costs associated with the maintenance model are inspection cost, basic maintenance cost, replacement cost and failure cost. Assumed cost parameters are listed in appendix. This analysis will give insight into all the associated costs. The simulation results, showing the relation between inspection rate and associated costs, are shown in Fig. 5-13. The following observations can be made out of the simulation results.

1. Failure cost decreases exponentially and then increases as the inspection rate of D1 increases and decreases exponentially as the inspection rate of D2 and D3 increases. This scenario can be observed in Fig 5, 8 and 11 respectively.
2. Maintenance cost first decreases and then increases with inspection rate of D1. Where as it increases and stays at constant value at higher inspection rate of D2 & D3. Fig. 6, 9 and 12 shows the variation of maintenance cost with inspection rate of D1, D2 and D3 respectively.
3. The optimal region of inspection rate of D1 that will minimize the total cost is 0.5-1 per year.
4. Maintenance of the device at its stage D1, beyond the optimal region is not useful.
5. Fig. 10 and 13 shows that the total cost is minimum at high inspection rate of D2 and D3 respectively.

Finally, results suggest that small inspection rate of D1 and high inspection rate of D2 and D3 will lead to cost effective maintenance. The model helps in allocating the available resources towards maintenance of the device and finds its importance in long term planning purposes.

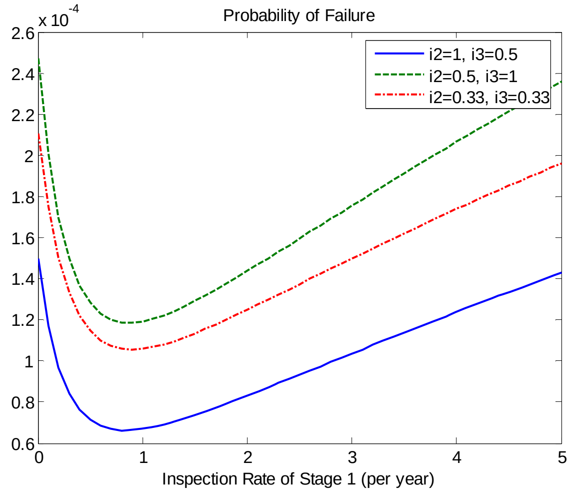


Fig 2: Failure Probability vs. Inspection rate of stage 1

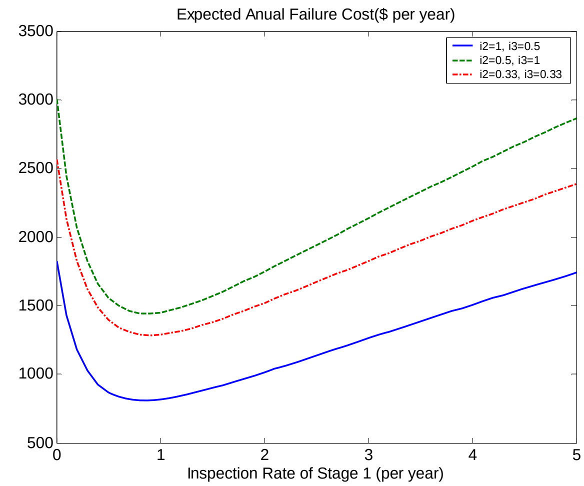


Fig 5: Failure cost vs. Inspection rate of stage 1

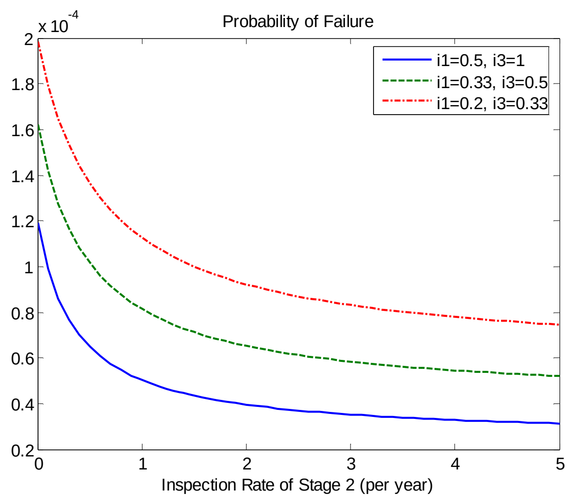


Fig 3: Failure Probability vs. Inspection rate of stage 2

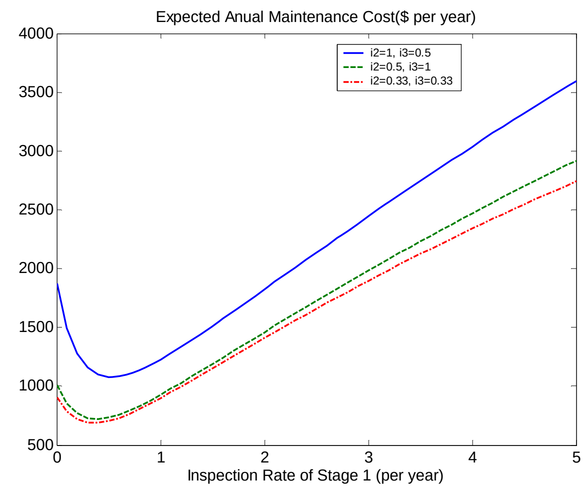


Fig 6: Maintenance cost vs. Inspection rate of stage 1

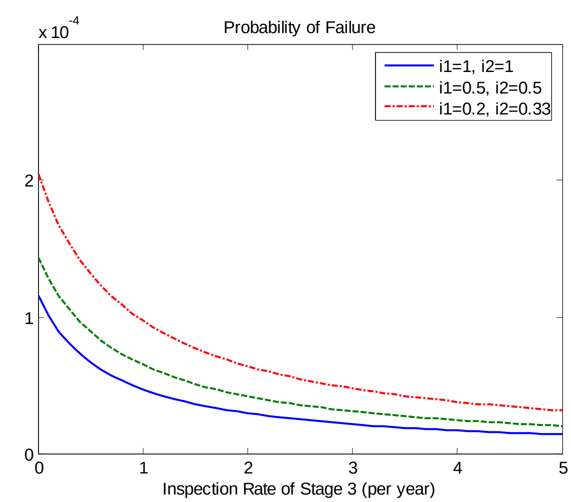


Fig 4: Failure Probability vs. Inspection rate of stage 3

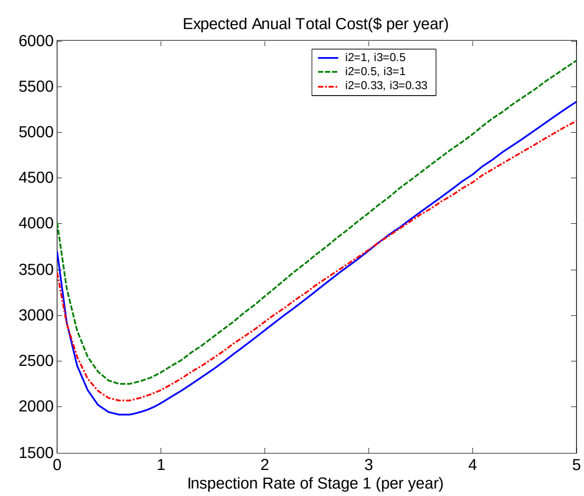


Fig 7: Total cost vs. Inspection rate of stage 1

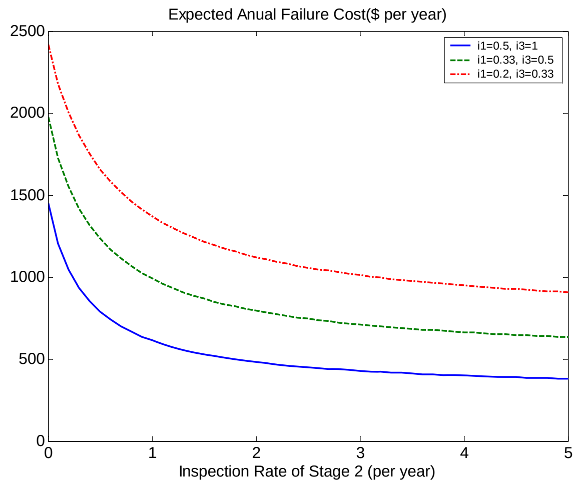


Fig 8: Failure cost vs. Inspection rate of stage 2

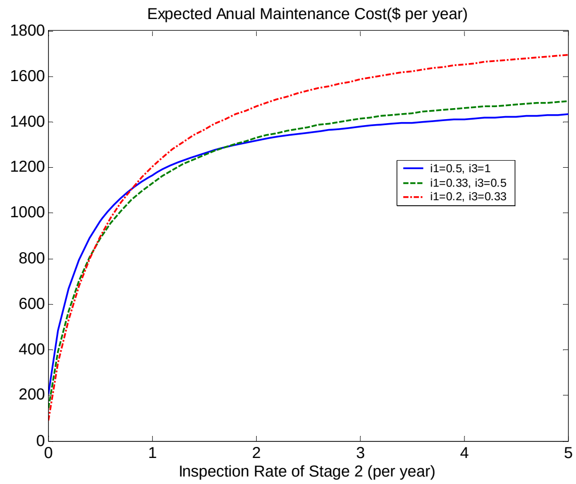


Fig 9: Maintenance cost vs. Inspection rate of stage 2

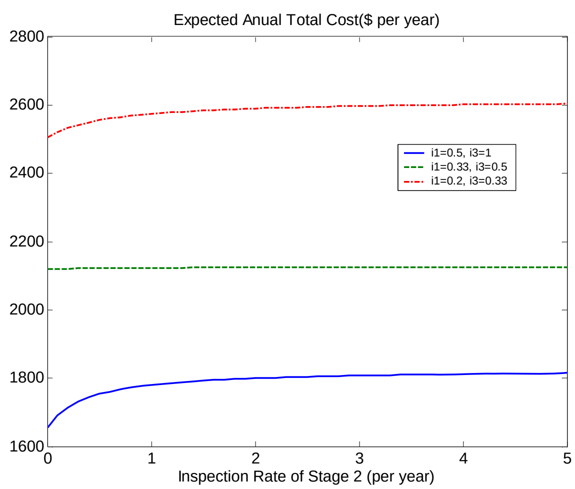


Fig 10: Total cost vs. Inspection rate of stage 2

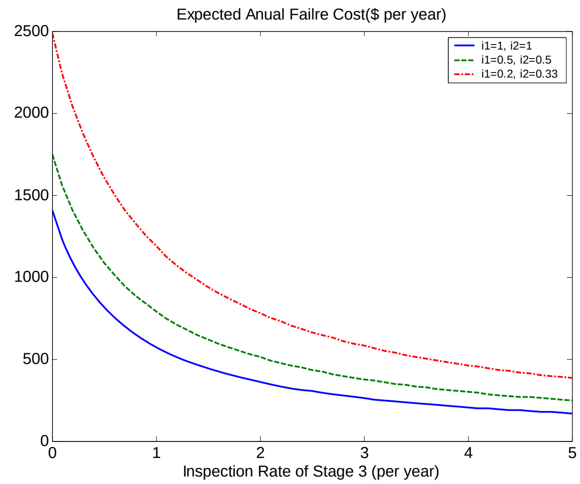


Fig 11: Failure cost vs. Inspection rate of stage 3

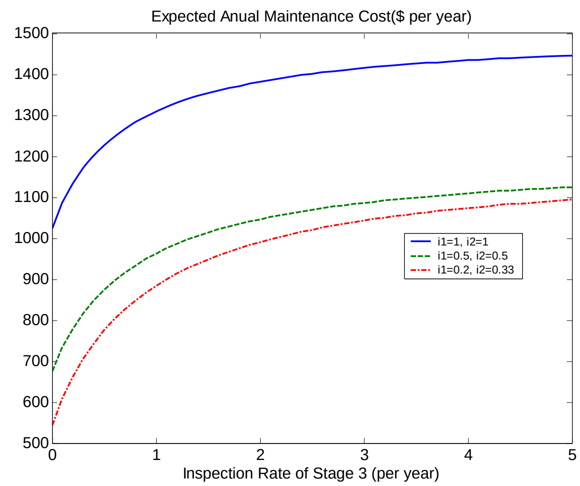


Fig 12: Maintenance cost vs. Inspection rate of stage 3

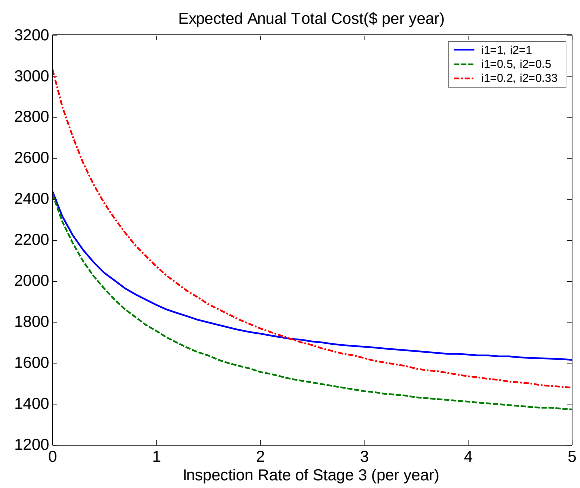


Fig 13: Total cost vs. Inspection rate of stage 3

V. EQUIVALENT MODEL FOR MATHEMATICAL ANALYSIS

In order to check the validity of the maintenance model presented in Fig. 1, it is necessary to introduce an equivalent model. Fig. 14 shows the equivalent model with 3 discrete stages representing the deterioration process of the breaker. Assume that maintenance is implemented at every inspection, maintenance and inspection rate of each stage is considered to be an equivalent repair rate.

Let D1: stage 1
D2: stage 2, minor deterioration
D3: stage 3, major deterioration
F: failure stage

y_1 = mean time in stage 1 (year)
y_2 = mean time in stage 2 (year)
y_3 = mean time in stage 3 (year)
μ_{21} = repair rate from stage 2 to 1 (/year)
μ_{32} = repair rate from stage 3 to 2 (/year)
μ_{31} = repair rate from stage 3 to 1 (/year)
μ_F = repair rate (/year).

Transition rate from stage 1 to 3 (λ_{13}) is introduced to describe an imperfect inspection of stage 1. This accounts for the probability that inspection of stage 1 might cause the system to transit to stage 3.

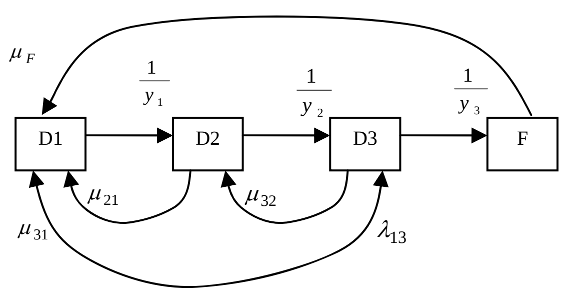


Fig. 14: Equivalent Maintenance Model

The mathematical analyses are presented in the next section using the steady state probability calculations [6]. The analyses cover both the probability of failure and cost analysis. The mathematical equations will be used to verify the simulation results presented in previous sections. Steady state probability calculations are presented in appendix.

A. Probability of failure analysis

Probability of failure can be expressed as the function of Mean Time to First Failure (MTTFF) and the repair rate (μ_F). Let T_0 = life time without maintenance and T_E = extended life time with maintenance. Then,

$$T_0 = y_1 + y_2 + y_3 \tag{1}$$

$$\begin{aligned} T_E = &(\mu_{21}\mu_{31} + \mu_{21}\mu_{32} + \mu_{21}\lambda_{13} + \mu_{32}\lambda_{13}) y_1 y_2 y_3 + \mu_{21} y_1 y_2 \\ &+ \mu_{31} y_2 y_3 + \mu_{31} y_1 y_3 + \mu_{32} y_2 y_3 + \lambda_{13} y_1 y_3 \end{aligned} \tag{2}$$

$$MTTFF = \frac{T_0 + T_E}{1 + \lambda_{13} y_1 + \lambda_{13}\mu_{21} y_1 y_2} \tag{3}$$

$$P_F = \frac{T_R}{T_R + MTTFF}, \tag{4}$$

and $T_R = \dfrac{1}{\mu_F}$ (5)

As mentioned in section III, the model parameter that is of interest, is the inspection rate at each stage. Following subsections are devoted to analyzing the relationships between the inspection rate of each stage and the probability of failure.

1) Inspection rate of stage 1

Increasing inspection rate of stage 1 increases the repair rate from stage 1 to stage 3 (λ_{13}). This results in decreasing failure probability as the denominator is higher than the numerator of (4). However, at higher inspection rate of stage 1, the numerator becomes more predominant than the denominator and hence the failure probability may increase. This result is observed in Fig 2. It is quite reasonable that if the device is in good condition, too much maintenance may decrease the life time.

2) Inspection rate of stage 2

Inspection rate of stage 2 results in increasing repair rate from stage 2 to 1, μ_{21}. Assuming that this repair rate is very high,

$$P_F \approx \frac{T_R \lambda_{13}\mu_{21} y_1 y_2}{T_R \lambda_{13}\mu_{21} y_1 y_2 + (\mu_{31} + \mu_{32} + \lambda_{13})\mu_{21} y_1 y_2 y_3} \tag{6}$$

It can be easily seen that the failure probability decreases with increase in repair rate (μ_{21}). This scenario is observed in Fig 3.

3) Inspection rate of stage 3

Inspection rate of stage 3 increases the repair rates from stage 2 to 3 (μ_{32}) and 1 (μ_{31}) respectively. These rates appear in the denominator of (4) and hence decrease the failure probability. This result is verified in Fig 4.

B. Cost analysis

The cost analyses include failure cost, maintenance cost, and total cost. Maintenance cost in this analysis includes inspection cost based on the assumption of the equivalent model that maintenance is implemented at every inspection. These equations will explain the simulation results in Fig 5-13.

The transitional probability matrix and resulting steady state probability are derived in appendix.

Let FC = repair cost after failure (dollar/time)
MC = maintenance cost (dollar/time)
P(i) = steady state probability of stage i; i=1,2, or 3
C_F = expected annual failure cost (dollar/year)
C_M = expected annual maintenance cost (dollar/year)
C_T = expected annual total cost (dollar/year)
T_R = repair time (year)

1) Failure Cost Analysis

The expected failure cost per year is, C_F = FC × frequency of failure which is equal to

$$C_F = FC \times (P_F \mu_F) = \frac{FC}{T_R + MTTFF} \tag{7}$$

It can be observed that without any maintenance, $C_F = FC/(T_R + T_0)$ is the highest possible value. If we assume that $\lambda_{13} << \lambda_{12}$ and $\lambda_{13} << \lambda_{23}$, then MTTFF will be higher and C_F will decrease as we increase repair rate of any stages (μ_{12}, μ_{31} or μ_{32}). On the other hand, if λ_{12} and λ_{13} are close to each other ($\lambda_{12}/\lambda_{13} \approx 1$), then MTTFF is possibly small. If MTTFF is small relative to T_R, then C_F will converge to, $C_F = FC/T_R$.

From Probability of failure analysis, P_F always decreases with maintenance as long as the probability of transferring from stage 1 to 3 is not high which is usually true. Therefore, failure cost will reduce to a constant value as inspection rate of any stage increases. This conclusion is verified by simulation results in Fig 5, 8, and 11.

2) Maintenance Cost Analysis

The expected maintenance cost per year is, C_M = MC × frequency of maintenance, which is equal to

$$C_M = MC \times (P(1)\lambda_{13} + P(2)\mu_{21} + P(3)(\mu_{31} + \mu_{32})) \tag{8}$$

Maintenance cost depends on repair rate of stage 2 and 3, if the probability of transferring from stage 1 to 3 is very small. In such case, it will increase from zero to some constant value. This is verified by simulation results in Fig 9 and 12. However, when inspection rate of D1 increases (probability of transferring from stage 1 to 3 is higher), maintenance cost could increase to infinity. It might be possible that the breaker condition gets even worse with every inspection and maintenance. Also, note that the maintenance cost includes the cost of inspection, which will increase with each inspection, resulting in increasing maintenance cost. This is verified by the simulation result in Fig 6.

3) Total Cost Analysis

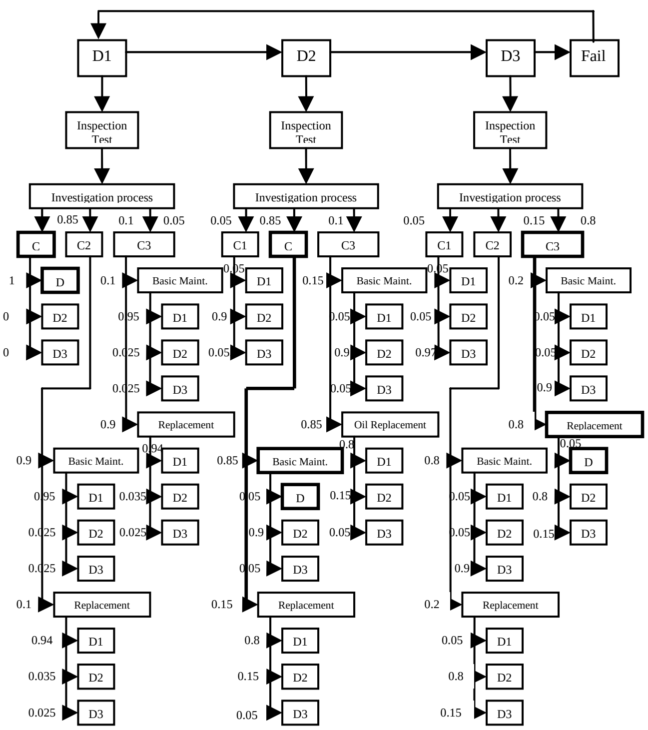
Total cost analysis gives an overall picture of relation between frequency of inspection rates and the associated cost. It can be observed from (7) and (8) that the failure cost dominates total cost at small inspection rates while maintenance cost dominates total cost at high inspection rate. The optimal value of the total cost will be the region with low inspection rate of stage 1 and high inspection rate of stage 2 and 3. The simulation results in Fig 7, 10 and 13 supports this conclusion.

VI. Conclusion

Sensitivity analysis of the probabilistic maintenance model introduced earlier for circuit breaker is done in this paper. The analysis covers the probability of failure, failure cost, maintenance cost and total cost. Simulation results from MATLAB are corroborated with mathematical equations of the equivalent model. The paper suggests that the optimal value of total cost is obtained with low inspection in stage 1 and high inspection in stage 2 and stage 3. The model finds its importance in long-term planning and allocation of resources over the life time of the breaker.

VII. Appendix

A. Model parameters



B. Cost parameters

Inspection cost = 100 \$
Basic maintenance cost = 1,000 \$
Replacement cost = 10,000 \$
Failure cost = 100,000 \$
Mean time in D1 = 12 years
Mean time in D2 = 9 years
Mean time in D3 = 4 years

C. Steady state probability calculations

Using frequency balance approach, steady state probability is calculated from

$$P = \begin{bmatrix} 1 & 1 & 1 & 1 \\ \frac{1}{y_1} & -\left(\mu_{21} + \frac{1}{y_2}\right) & \mu_{32} & 0 \\ \lambda_{13} & \frac{1}{y_2} & -\left(\mu_{31} + \mu_{32} + \frac{1}{y_3}\right) & 0 \\ 0 & 0 & \frac{1}{y_3} & -\mu_F \end{bmatrix}^{-1} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{9}$$

$$\begin{aligned} \det(P) &= -\frac{\mu_F}{y_1 y_2 y_3}\left[\frac{1}{\mu_F}\left(1 + y_1\lambda_{13} + y_1 y_2 \mu_{21}\lambda_{13}\right) + MTTFF\left(1 + y_1\lambda_{13} + y_1 y_2 \mu_{21}\lambda_{13}\right)\right] \\ &= -\frac{\mu_F}{y_1 y_2 y_3}\left(T_R + MTTFF\right)\left(1 + y_1\lambda_{13} + y_1 y_2 \mu_{21}\lambda_{13}\right) \end{aligned} \tag{10}$$

$$MTTFF = \frac{T_0 + T_E}{1 + \lambda_{13} y_1 + \lambda_{13}\mu_{21} y_1 y_2}, \tag{11}$$

$$T_0 = y_1 + y_2 + y_3\,, \tag{12}$$

$$T_E = (\mu_{21}\mu_{31} + \mu_{21}\mu_{32} + \mu_{21}\lambda_{13} + \mu_{32}\lambda_{13})y_1 y_2 y_3 + \mu_{21}y_1 y_2 + \mu_{31}y_2 y_3 + \mu_{31}y_1 y_3 + \mu_{32}y_2 y_3 + \lambda_{13}y_1 y_3\,, \tag{13}$$

$$T_R = 1/\mu_F \tag{14}$$

Then, the steady state probabilities are

$$P = \frac{1}{(T_R + MTTFF)}\begin{bmatrix} \dfrac{y_1 + y_1 y_2 \mu_{21} + y_1 y_3 \mu_{31} + y_1 y_2 y_3 \mu_{21}\mu_{31} + y_1 y_2 y_3 \mu_{21}\mu_{32}}{(1 + \lambda_{13}y_1 + \lambda_{13}\mu_{21}y_1 y_2)} \\ \dfrac{y_2 + y_2 y_3 \mu_{31} + y_2 y_3 \mu_{32} + y_1 y_2 y_3 \lambda_{13}\mu_{32}}{(1 + \lambda_{13}y_1 + \lambda_{13}\mu_{21}y_1 y_2)} \\ y_3 \\ T_R \end{bmatrix} \tag{15}$$

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