A NEW ATP ADD-ON FOR MODELING INTERNAL FAULTS IN POWER TRANSFORMERS

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INTRODUCTION

The more sophisticated a protection algorithm, the more information on both steady-state and transient behavior of a protected element required for proper designing, setting and testing of a given relay.

In the case of a power transformer, the most important phenomena to be modeled from the standpoint of protective relaying are:

- magnetizing inrush including initial, recovery and sympathetic inrush as well as out-of-step synchronization of a connected generator,
- stationary overexcitation of the core due to short-term steady-state overvoltage and/or frequency reduction,
- internal short-circuits including turn-to-turn, inter-winding and earth faults,
- external faults combined with transformer and Current Transformers (CTs) ratio mismatch, on-load tap changer operation, and saturation of the CTs,
- double contingency events such as switching-in a faulted transformer or internal fault occurring in course of an external fault, etc.

Since field recordings of transformers abnormal conditions, especially for internal faults, are seldom available, the information needed for investigation of protective systems may be achieved exclusively by means of digital simulation.

The main directions in the computer modeling for analysis of transformers are classified as follows[1]:

- Self and mutual inductances. The approach is commonly used in transient short-circuit calculations since adopted as the transformer model in EMTP-ATP[2]. The method uses accurate formulae for calculation of self and mutual inductances between the windings. The presence of an iron core, however, makes the values of inductances close one to another which results in ill-conditioned equations. This problem has been efficiently solved by subtracting the common flux when computing the inductances.
- Leakage inductances. This model represents adequately the leakage inductances of a transformer but shows difficulties in representing properly the iron core.
- Principle of duality. This approach deals accurately with the iron core, but the leakage inductances, in turn, cannot be modeled properly.
- Measurements. This family of methods focuses on representing a transformer as a terminal equivalent in a wide spectrum of frequencies. Methods of this group match the parameters of an assumed structure of a model with the experimental frequency response of a tested transformer. Weak basis for generalization is the major drawback of those methods.
- Electro-magnetic fields. This group of methods use three-dimensional finite element algorithms for analysis of electro-magnetic fields in a power transformer. The approach is very accurate and complete but rather design oriented due to a very heavy computational burden.

Most of the above models have been primarily developed as global models - they give a terminal equivalent of the device. In order to simulate disturbances such as internal faults, one needs a model with inner nodes in its windings rather than very accurate representation of the core or emulation of the frequency response of a transformer.

This paper presents a model of a power transformer with an internal fault as well as the software implementation of this model. The approach taken in this paper is based on the BCTRAN[2] procedure of ATP. The terminal equivalent of a transformer delivered by BCTRAN is rearranged using the custom-built software to incorporate model of an internal fault. The resulting BCTRAN-like data file is then processed by ATP for simulation.

First, a terminal equivalent of a transformer is presented. Second, the modifications are described that yield the internal fault model. Third, the developed ATP add-on software is presented. The numerical example is given to illustrate the simulation method.

TERMINAL EQUIVALENT OF A TRANSFORMER

The terminal equivalent of a transformer meets the needs of modeling external faults, magnetizing inrush and overexcitation conditions. It is also a starting point for modeling internal faults in a power transformer.

Neglecting the core nonlinearities, a set of mutually coupled linear RL coils is commonly used to represent a transformer[3]. Thus, the terminal equivalent in the time domain is given as:

$$[v] = [R][i] + [L]\frac{d}{dt}[i] \quad (1a)$$

or using the inverse matrix [A] = $[L]^{-1}$:

$$\frac{d}{dt}[i] = [A][v] - [A][R][i] \quad (1b)$$

where:

[v], [i] vectors of the terminal voltages and currents, respectively, of a considered transformer;

[R], [L], [A] parameters of the model.

The alternatives (1a) and (1b) although mathematically the same, differ when the numerical issues are involved. The selection between them depends on the test data available for calculation of the parameters.

The following methods for calculating the parameters of the model (1) are commonly used:

a. Steady state excitation and short-circuit test data in the positive- and zero-sequence domains[2,3].
b. Time domain tests using the least error squares technique for fitting the parameters of the model to the excitation (no-load) and load waveforms[4].
c. Analytical approach involving analysis of electromagnetic fields[1,5].

The steady state short-circuit and excitation tests are practical and therefore used in this paper for developing the terminal equivalent of a transformer.

Single-Phase Transformers

Consider a single-phase *N*-winding transformer. In the steady state, the transformer is modeled by the phasor equation:

$$[V] = [Z][I] \quad (2)$$

The diagonal elements of [Z] are obtained from the excitation test data using the well-known relation[3]:

$$Z_{ii} = f\left(I_{exct\,i}, P_{Fe\,i}\right) \quad (3)$$

where:

$I_{exct\ i}$ excitation current during the excitation test for the i-th winding,

$P_{Fe\ i}$ core losses during the test.

The off-diagonal elements of [Z] are calculated from the short-circuit test data:

$$Z_{ik} = f\left(P_{Cu\,ik}, X_{ik}^{short}\right) \quad (4)$$

where:

(*i*,*k*) considered pair of windings in the short-circuit test,

$P_{Cu\ ik}$ copper losses during the test,

X_{ik}^{short} short circuit pu voltage.

The copper losses allow computing the winding resistance while the short-circuit voltage enables for computing the reactance.

Given the short circuit impedance Z_{ik}^{short} the mutual impedance is calculated as:

$$Z_{ik} = Z_{ki} = \sqrt{\left(Z_{ii} - Z_{ik}^{short}\right)Z_{kk}} \quad (5)$$

The sought matrices [R] and [L] in the basic model (1) are derived from the real and imaginary parts of [Z], respectively.

Three-Phase Transformers

Equation (5) holds true for three-phase transformers as well after the following manipulations[3]:

1. The current, voltage and impedance scalars become appropriate matrices:

$$i_k \to \begin{bmatrix} i_{kA} & i_{kB} & i_{kC} \end{bmatrix}^T \quad (6a)$$

$$v_k \to \begin{bmatrix} v_{kA} & v_{kB} & v_{kC} \end{bmatrix}^T \quad (6b)$$

$$Z_{ki} \to \begin{bmatrix} Z_{kis} & Z_{kim} & Z_{kim} \\ Z_{kim} & Z_{kis} & Z_{kim} \\ Z_{kim} & Z_{kim} & Z_{kis} \end{bmatrix} \quad (6c)$$

where:

A, B, C phase indices,

Z_{kis} self impedance between the windings *k* and *i* (i.e. the mutual impedance between the same phase of the two windings),

Z_{kim} mutual impedance between the windings *k* and *i* (i.e. the mutual impedance between different phases of the two windings).

2. The excitation and short-circuit tests are performed in both the positive sequence (direct test) and zero sequence (homopolar test) domains.

Equations (3)-(5) are applied separately for the zero- and positive-sequence test data. The obtained impedances are next re-calculated into the self and mutual quantities using the well known relations:

$$Z_s = \frac{1}{3}\left(Z_0 + 2Z_1\right), \qquad Z_m = \frac{1}{3}\left(Z_0 - Z_1\right) \quad (7)$$

Autotransformers and Three-Winding Transformers

Particular attention must be paid when applying this approach to autotransformers and three-winding transformers with at least one winding connected in DELTA.

Let us first consider a three-winding (H, X and Y) transformer with its tertiary winding Y connected in DELTA. Since the DELTA winding acts as a short-circuit for the zero sequence current, during a homopolar short-circuit test between the windings H and X there are actually as many as two windings shorted (X and Y). Therefore[3]:

$$Z_{HX}^{short} = Z_H + \frac{Z_X Z_Y}{Z_X + Z_Y} \quad \text{in pu values} \qquad (8a)$$

$$Z_{HY}^{short} = Z_H + Z_Y \quad \text{in pu values} \qquad (8b)$$

$$Z_{XY}^{short} = Z_X + Z_Y \quad \text{in pu values} \qquad (8c)$$

Since only the magnitudes $|Z^{short}|$ in (8) are obtained from the short-circuit tests, equations (8) must be solved numerically together with the copper losses equations in order to find Z_H, Z_X and Z_Y.

Once found, the values of winding impedances Z_H, Z_X and Z_Y should be re-calculated into the input data for the basic algorithm (3)-(7) as follows:

$$Z_{HX}^{short} = Z_H + Z_X \quad \text{in pu values} \qquad (9a)$$

$$Z_{HY}^{short} = Z_H + Z_Y \quad \text{in pu values} \qquad (9b)$$

$$Z_{XY}^{short} = Z_X + Z_Y \quad \text{in pu values} \qquad (9c)$$

The same problem arises when considering a homopolar excitation test for a three-winding transformer with a DELTA-connected winding. Since the DELTA winding acts as a short-circuit for the zero sequence current, the test becomes actually a short-circuit test instead of an excitation (open circuit) test. Therefore, the winding connected in DELTA must be opened during the test, otherwise the excitation parameters shell be neglected or assumed to be the same as for the positive sequence[2,3].

An autotransformer in this approach is treated as an appropriately connected multi-transformer. Let us analyze an autotransformer with the tertiary winding as in Fig.1. The autotransformer is represented by three windings 1, 2 and 3, for which the short-circuit data are re-calculated as follows[3]:

$$Z_{12}^{short} = Z_{HX}^{short}\left(\frac{V_H}{V_H - V_X}\right)^2, \quad Z_{23}^{short} = Z_{XY}^{short} \qquad (10a)$$

$$Z_{13}^{short} = Z_{HX}^{short}\frac{V_H V_X}{(V_H - V_X)^2} + \ldots$$

$$\ldots + Z_{HY}^{short}\frac{V_H}{V_H - V_X} - Z_{XY}^{short}\frac{V_X}{V_H - V_X} \qquad (10b)$$

The values given by (10) are next used in the basic algorithm (3)-(7).

Impedance Approach vs. Admittance Approach

When the excitation current is neglected or intended to be represented by a more sophisticated model, the matrix [L] cannot be obtained from the short-circuit data alone. Consequently, the model (1a) is no longer useful. In such a case, the alternative model (1b) is recommended.

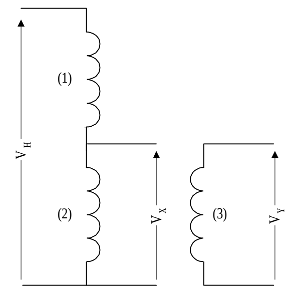

Figure 1. Autotransformer with a tertiary winding.

The matrix [A] in (1) is originated as[3]:

$$[A] = j\omega[Y] \qquad (11)$$

while the matrix [Y] is formed as:

$$Y_{ik} = Y_{ik}^{r} \qquad k < N \qquad (12a)$$

$$Y_{iN} = -\sum_{k=1}^{N} Y_{ik}^{r} \qquad i < N \qquad (12b)$$

$$Y_{NN} = -\sum_{k=1}^{N-1} Y_{iN}^{r} \qquad (12c)$$

where: $$\left[Y^r\right] = \left[Z^r\right]^{-1} \qquad (13)$$

and $$Z_{ii}^{r} = Z_{iN}^{short} \qquad (14a)$$

$$Z_{ik}^{r} = \frac{1}{2}\left(Z_{iN}^{short} + Z_{kN}^{short} - Z_{ik}^{short}\right) \qquad (14b)$$

The admittance approach (1b), (11)-(14) is based on the short-circuit data only (neglecting the excitation test data) and shows better numerical accuracy than (1a) by avoiding ill-conditioned relations. The excitation model may be added to (11b) as a separate linear or nonlinear shunt branch.

MODELING INTERNAL FAULTS

The key assumptions of our modeling method are:

1. The transformer is given as a terminal equivalent in the form of the pre-computed self and mutual impedances (see the previous section).
2. A winding with an internal fault is divided between two (winding-to-ground faults) or three (turn-to-turn faults) sub-coils.

The basic problem of this approach is how to calculate the self and mutual impedances between the created sub-coils and the rest of the healthy coils. The problem is solved by using the principles of: consistency, leakage, and proportionality. This approach may be supported by the leakage factors computed from the winding geometry[5].

For simplicity of notation let us consider a two-winding transformer. The pre-computed matrices [R] and [L] for a transformer shown in Fig.2a are:

$$[R] = \begin{bmatrix} R_1 & & 0 \\ & \ddots & \\ 0 & & R_6 \end{bmatrix} \quad (15a)$$

$$[L] = \begin{bmatrix} L_1 & L_{12} & \cdots & L_{16} \\ L_{21} & L_2 & \cdots & L_{26} \\ \vdots & \vdots & \ddots & \vdots \\ L_{61} & L_{62} & \cdots & L_6 \end{bmatrix} \quad (15b)$$

Winding-to-Ground Internal Faults

Assume the winding (1) suffers an internal winding-to-ground fault (Fig.2b). A single inner node is thus created which divides the coil (1) between two sub-coils, say a and b (Fig.2b) and the matrices [R] and [L] become:

$$[R] = \left[\begin{array}{cc|ccc} R_a & 0 & & & 0 \\ 0 & R_b & & & \\ \hline & & R_2 & & 0 \\ & & & \ddots & \\ 0 & & 0 & & R_6 \end{array}\right] \quad (16a)$$

$$[L] = \left[\begin{array}{cc|ccccc} L_a & L_{ab} & L_{a2} & L_{a3} & L_{a4} & L_{a5} & L_{a6} \\ L_{ba} & L_b & L_{b2} & L_{b3} & L_{b4} & L_{b5} & L_{b6} \\ \hline L_{2a} & L_{2b} & L_2 & & & & \\ L_{3a} & L_{3b} & & \ddots & & & \\ L_{4a} & L_{4b} & & & \ddots & & \\ L_{5a} & L_{5b} & & & & \ddots & \\ L_{6a} & L_{6b} & & & & & L_6 \end{array}\right] \quad (16b)$$

and the problem reduces to computation of the indicated portions of the matrices (16).

For the resistances, a simple proportionality principle is physically obvious, thus:

$$R_a = \frac{n_a}{n_1} R_1 \qquad R_b = \frac{n_b}{n_1} R_1 \quad (17)$$

where: n_a, n_b number of turns in the sub-coils a and b respectively, $n_a + n_b = n_1$.

For computation of the self and mutual inductances between the sub-coils a and b the following rules are used[5]:

1. Consistency: $L_a + 2L_{ab} + L_b = L_1$ (18a)

2. Leakage: $\delta_{ab} = 1 - \dfrac{L_{ab}^2}{L_a L_b}$ (18b)

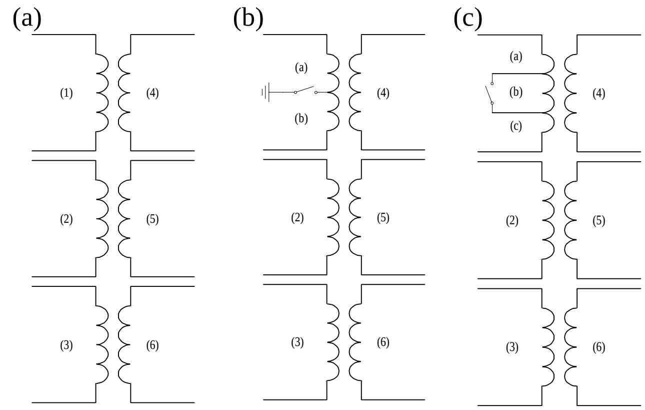

Figure 2. Three-phase two-winding transformer without (a), with ground (b) and turn (c) an internal fault.

3. Proportionality: $\dfrac{L_a}{L_b} = \left(\dfrac{n_a}{n_b}\right)^2$ (18c)

Assuming δ_{ab} known (it is to be either computed from the winding geometry or assumed as a parameter) and denoting $k = n_a/n_b$, equations (18) yield:

$$L_a = \frac{L_1}{\dfrac{1}{k^2} + \dfrac{2\sqrt{1-\delta_{ab}}}{k} + 1} \quad (19a)$$

$$L_b = \frac{L_1}{k^2 + 2k\sqrt{1-\delta_{ab}} + 1} \quad (19b)$$

$$L_{ab} = \frac{L_1\sqrt{1-\delta_{ab}}}{\left(k + \dfrac{1}{k}\right) + 2\sqrt{1-\delta_{ab}}} \quad (19c)$$

For computation of the mutual inductances between the sub-coils a and b and the rest of the coils (2..6) one should distinguish two cases[5]:

1. the considered i-th coil is wound on the same leg as the sub-coils a and b:

$$L_{ai} = L_{ia} = L_{1i}\sqrt{\varepsilon}\sqrt{\frac{L_a}{L_1}}\sqrt{1 + \frac{1-\varepsilon}{\varepsilon}\frac{L_1 L_i}{L_{1i}^2}} \quad (20a)$$

$$L_{bi} = L_{ib} = L_{1i} - L_{ai} \quad (20b)$$

where: $\varepsilon = \dfrac{\delta_{ai}}{\delta_{1i}}$ (20c)

2. the considered i-th coil is wound on a different leg than the sub-coils a and b:

$$L_{ai} = L_{ia} = \frac{k}{1+k} L_{1i}\,,\; L_{bi} = L_{ib} = \frac{1}{1+k} L_{1i} \quad (21)$$

Turn-to-Turn Internal Faults

Consider the winding (1) suffering a turn-to-turn fault. Two inner nodes must be created that divide the coil between three sub-coils a, b and c (Fig.2c). Consequently, the matrices [R] and [L] become:

$$[R] = \left[\begin{array}{ccc|ccc} R_a & & 0 & & & \\ & R_b & & & 0 & \\ 0 & & R_c & & & \\ \hline & & & R_2 & & 0 \\ & 0 & & & \ddots & \\ & & & 0 & & R_6 \end{array}\right] \qquad (22a)$$

$$[L] = \left[\begin{array}{ccc|ccc} L_a & L_{ab} & L_{ac} & L_{a2} & \cdots & L_{a6} \\ L_{ba} & L_b & L_{bc} & L_{b2} & \cdots & L_{b6} \\ L_{ca} & L_{cb} & L_c & L_{c2} & \cdots & L_{c6} \\ \hline L_{2a} & L_{2b} & L_{2c} & L_2 & \cdots & L_{26} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ L_{6a} & L_{6b} & L_{6c} & L_{62} & \cdots & L_6 \end{array}\right] \qquad (22b)$$

and again we need to compute certain portions of the matrices (22).

The solution is obtained in a similar way as when breaking a winding coil into two sub-coils[5].

Leakage Factors

Ideally, the leakage factor can be calculated using the core and winding geometrical data. In this paper, the winding geometry is assumed to be unavailable. Consequently, the leakage factors are treated as parameters in our approach.

For the leakage factor we recommend to use the average leakage for a given core leg. The leakage factor between any pair of coils (*i*,*j*) may be computed from the terminal equivalent of a transformer using the elementary equation:

$$\delta_{ij} = 1 - \frac{L_{ij}^{\,2}}{L_i L_j} \qquad (23)$$

For an unknown ratio of two leakage factors we recommend to use the value of 1.0. The numerical examples show that the unknown parameters, if close to the suggested average values (23), have little influence on the obtained simulation results. Thus, the outlined approach is applicable at least for protective relaying studies.

IMPLEMENTATION

The aforedescribed model for three-phase multi-winding transformers and autotransformers with internal winding-to-ground and turn-to-turn faults has been implemented as a stand-alone executable add-on to ATP. The Transformer Fault Analysis Assistant (TFAA)[6] works in two steps.

First (Fig.3), the software enables the user to enter the transformer's data and prepares the input file for ATP's BCTRAN (*.in file). Once this step is completed, ATP is invoked and the BCTRAN output file (*.pch file) is created. The latter file contains the coupled-RL model of a sound transformer.

Second (Fig.3), the software reads the *.pch file, enables the user to specify the fault parameters (type, location, leakage factors, etc.) and generates the ATP includable file (*.lib file) that may be $INCLUDEd[2] into any ATP model. The *.lib file contains the coupled-RL model of a faulted transformer.

NUMERICAL EXAMPLE

An autotransformer is used in this numerical example. Table 1 gathers the data used.

Verification of the Terminal Equivalent

The obtained terminal equivalent of the autotransformer has been validated by comparing the short-circuit and excitation physical tests and simulation results. As shown in Tables 2 and 3, the terminal equivalent is sufficiently accurate.

Sample of Evolving Internal Fault

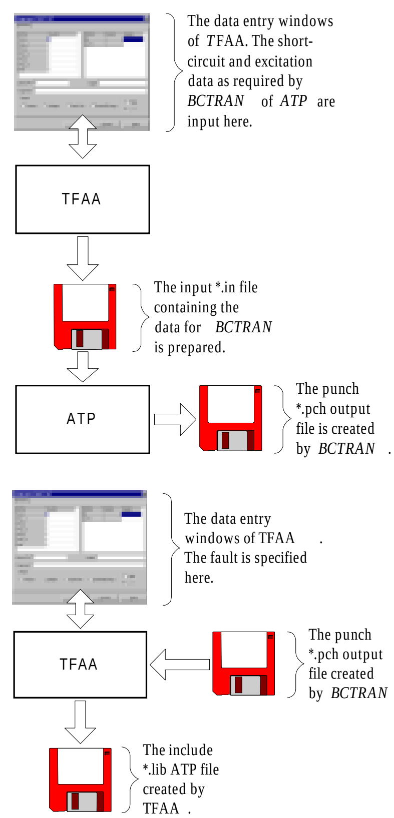

Figure 3. Illustration of using TFAA.

At t = 25msec a turn-to-turn fault occurs in the DELTA-connected tertiary winding that involves 5% of its turns. Ten msec later, as the fault unfolds, a new fault path is established between the faulted part of the tertiary winding and the core. Twenty five msec after that, yet another fault path emerges between the faulted winding and the secondary winding on the same core leg. Fig.4 shows the terminal currents during this sample evolving fault.

CONCLUSIONS

This paper presents a transformer model and its software implementation capable of modeling internal winding-to-ground and turn-to-turn internal faults in three-phase multi-winding transformers and autotransformers.

The method represents a transformer by coupled-RL coils and adds an internal fault model by sub-dividing the windings into sub-coils and appropriately calculating the parameters of the new coils.

The stand-alone executable software has been developed that supports both the data preparation for the BCTRAN of ATP as well as creation of the transformer model with an internal fault.

The software can be used as an add-on with ATP or in conjunction with advanced modeling and testing tools for short-circuit related activities[7] such as protective relay setting, testing and evaluating.

Table 1. The data of the autotransformer.

	Voltage [kV]	Power [MVA]	Resistance [ohm]
H	345 GRD.Y	448	1.0961 / 3 phases
X	138 GRD.Y	448	0.3335 / 3 phases
Y	13.09 DELTA	107.5	0.0161 / 3 phases
	Short-Circuit Tests		
H-X	12.04% @ 448MVA		
H-Y	32.32% @ 448MVA		
X-Y	17.95% @ 448MVA		
	Excitation Test		
	I = 0.038%	P = 77.45kW	from H @ 100% V

Table 2. Validation of the excitation data.

Quantity	Test	Sim.	Error [%]
Excitation current, winding H [A]	0.2848	0.2830	0.62
Excitation losses [kW]	77.45	71.71	7.4
Secondary voltage [kV]	138	138.00	0.0
Tertiary voltage [kV]	13.05	13.07	0.1

Table 3. Validation of the short-circuit data.

Test	Test	Sim.	Error [%]
X shorted, Y opened, 12.04% of the rated V_H applied - the current at H reads [A]:	749.72	749.83	0.02
Y shorted, X opened, 32.32% of the rated V_H applied - the current at H reads [A]:	749.72	749.94	0.03
Y shorted, H opened, 17.95% of the rated V_X applied - the current at X reads [A]:	1874.3	1874.7	0.02

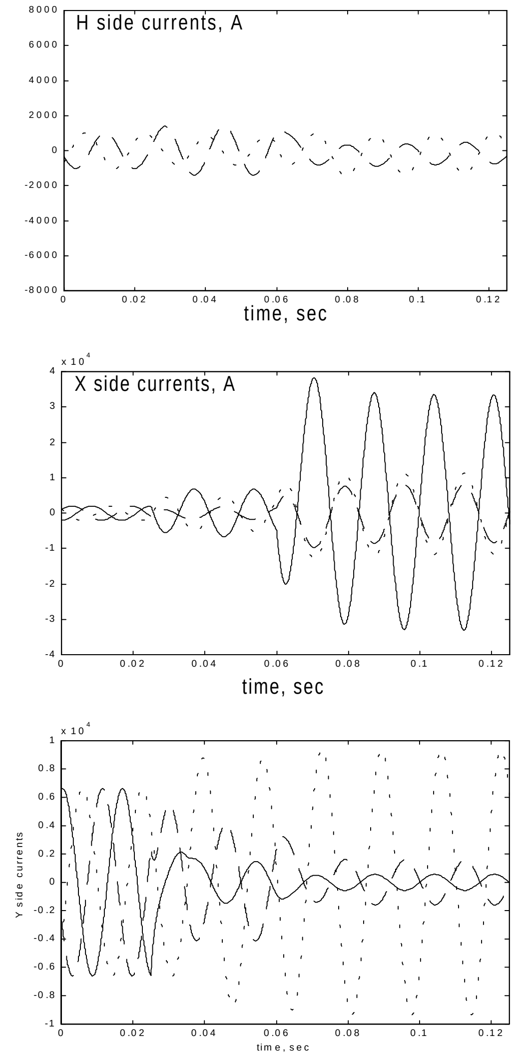

Figure 4. Currents during a sample evolving fault.

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