DIGITAL METERING OF ACTIVE AND REACTIVE POWER IN NONSINUSOIDAL CONDITIONS USING BILINEAR FORMS OF VOLTAGE AND CURRENT SAMPLES

B. Peruničić and S. Levi
University of Sarajevo

M. Kezunović and E. Šoljanin
Texas A&M University

The sequences of regularly spaced voltage and curent samples form the basis for digital metering of active power. Many algorithms for measurement of active power and reactive power were proposed and implemented up to now. Some of them are more used then the others, but none, as yet, is recognized as the most suitable one for the purpose.

One of the reasons for such a state of the art is that discussions on the definition of active and reactive power in nonsinusoidal conditions are still continuing. For example, active power is sometimes undestood as an average of instantaneous power taken in the period, and sometimes as the average of the fundamental harmonic only. Even more diputable is the definition of the reactive power [1], [2], [3], [4].

The common feature of all currently used algorithms is that they provide the same value if the voltage and the current are pure sinusoids, but they results differ if the other harmonics are present. Therefore it is dificult to decide which one is the most suitable for the purpose.

One of the causes of such a state is that there are not yet digital algorithms for some definition of measurements. For example Kuster's inductive and reactive power can be obtained by solide state devices; there are no proposed way to calculate Budeanu's reactive power by an analog or a digital way. That makes dificult to predict their response for various signal content, so that algorithms may be compared.

The aim of this paper is to provide a theoretical base for the calculation of the response to the periodic signals, and to promote a new method for the design of new algorithms.

This method is based on a general expression, which matematicaly can be defined as a bilinear form of the signal samples. Some existing algorithms may be recognized as special cases of such a form, and the other have bilinear forms as their building blocs [9]. In [6] and [9] such forms were used to design the algorithms for line impedance measurement and power measurement, but under the assumptionn that signals are pure sinusoids.

In this paper the same forms are used for measurement of active and reactive power of the periodic signals. Since active and reactive power are often expressed in frequency domain, it is conve nient to define signal models in such a manner.

Signal models

In steady state condition voltage and current are periodic signals if the noise term is neglected. If active and reactive power are calculated by integration and multiplication, then it is posible to obtain exactly such values as the sum of active powers of all the harmonics or active and reactive power of the first harmonic.

However if the same algorithms are performed by multiplication and summation of samples, then meaningful results may be obtained only if the number of the harmonics is adjusted in accordance with Niquist rate [8]. This is the reason to adopt the following model of voltage and current samples:

$$v_n = \sum_{m=0}^{M} V_m \cos(ndm + \varphi_m), \quad \varphi_0 = 0 \tag{1}$$

$$i_n = \sum_{m=0}^{M} I_m \cos(ndm + \psi_m), \quad \psi_0 = 0 \tag{2}$$

Here n represents discrete time, $d = 2\pi/N$, and $M \leqslant 0{,}5N - 1$ (N is even). Since v_n and i_n are periodic series, with a period N, they can be represented with their Discrete Fourier Series (DFS) coefficients denoted $\tilde{V}(k)$ and $\tilde{I}(k)$. Their values are:

$$\tilde{V}(k) = \begin{cases} V_k \cdot e^{j\varphi_k}, & k = 0, 1, 2, \ldots, M \\ V_{N-k} \cdot e^{j\varphi_{N-k}}, & k = N-K, \ldots, N-1 \\ 0, & \text{other values of } k \end{cases} \tag{3}$$

$$\tilde{I}(k) = \begin{cases} I_k \cdot e^{j\psi_k}, & k = 0, 1, 2, \ldots, M \\ I_{N-k} \cdot e^{j\psi_{N-k}}, & k = N-M, \ldots, N-1 \\ 0, & \text{other values of } k \end{cases} \tag{4}$$

Algorithm model

The bilinear form is defined as: [6].

$$\widetilde{bf}(n) = \sum_{k=0}^{N-1} \sum_{m=0}^{N-1} h_{km} v_{n-k}\, i_{n-m} \tag{5}$$

The coefficients h_{km} can be seen as the weights attached to the product of voltage and current samples taken in discrete times k and m moments before the present moment n respectively. For example, the algorithm for active power calculation, described by the expression

$$P = \frac{1}{N} \sum_{k=0}^{N-1} u_{n-k}\, i_{n-k} \qquad (6)$$

can be recognized as a bilinear form having $h_{km} = \frac{1}{N}\delta\,(k-m)$. The construction of new algorithm can be viewed as a selection of a N x N weight matrix H, having as its elements h_{km}. In this paper the possible values of bilinear forms of periodic signals will be found, and the principles of the algorithms design will be given. The analysis and the design of the algorithms will be done in frequency domain, using the techniques of DFS.

The value of bilinear form of periodic samples

The product $v_n\ i_r$, where n and r are discrete times may be considered as a two-dimensional separable signal x_{nr}. If this signal passes through a linear two-dimensional FIR system having h_{km}; k, m = 0, 1, 2, . . . , N−1 as its impulse response, the value of the output y_{nr} will be:

$$y_{nr} = \sum_{k=0}^{N-1} \sum_{m=0}^{N-1} h_{km}\ x_{n-k,\,r-m} = \sum_{k=0}^{N-1} \sum_{m=0}^{N-1} h_{km}\, v_{n-k}\, i_{r-m} \qquad (7)$$

The two–dimensional DFS of the output, denoted $\tilde{Y}$ (p, q) will be

$$\tilde{Y}\,(p,q) = \tilde{H}\,(p,q)\ \tilde{V}\,(p)\,\tilde{I}\,(q)$$

Here H (p, q) is two–dimensional Discrete Fourire Transform of the real sequence h_{km}. It can be easily shown that it has the following properties:

$$\tilde{H}^{*}(p,q) = \tilde{H}\,(N-p,\,N-q) \qquad (8)$$

$$\tilde{H}\,(p+N,\,q) = \tilde{H}\,(p,\,q+N) = \tilde{H}\,(p+N,\,q+N) = \tilde{H}\,(p,q) \qquad (9)$$

Also, if H(p, q) is real

$$h_{km} = h_{mk} \qquad (10)$$

and if H (p, q) is imaginary

$$-h_{km} = h_{mk} \qquad (11)$$

The output can be expressed as an inverse DFS:

$$y_{nr} = \frac{1}{N^2} \sum_{p=0}^{N-1} \sum_{q=0}^{N-1} \tilde{H}\,(p,q)\ \tilde{V}\,(p)\,\tilde{I}\,(q)\ W_N^{-np}\,W_N^{-rq} \qquad (12)$$

If r = n, then

$$y_{nn} = \tilde{bf}\,(n) = \frac{1}{N^2} \sum_{p=0}^{N-1} \sum_{q=0}^{N-1} \tilde{H}\,(p,q)\ \tilde{V}\,(p)\,\tilde{I}\,(q)\ W_N^{-(p+q)n} \qquad (13)$$

Since $\tilde{bf}$ (n) is a periodic series, with a period N, it has DFS series coefficients $\tilde{BF}$ (k):

$$\tilde{BF}\,(k) = \frac{1}{N} \sum_{p=0}^{N-1} \tilde{H}\,(p,\,N+k-p)\ \tilde{V}\,(p)\,\tilde{I}\,(N+k-p) \qquad (14)$$

Then, bf (n) can be expressed as:

$$\tilde{bf}\,(n) = \frac{1}{N} \sum_{k=0}^{N-1} \tilde{BF}\,(k)\ W_N^{-kp} \qquad (15)$$

This expression is a start–point for the algorithm design.

The algorithm design

If the current and the voltage are periodic signals, active and reactive power, by all the known definitions are constant. So, any bilinear form must have a value independent on n in order to represent some algorithm for active or reactive power measurement. This condition imposes the following restrictions of $\tilde{BF}$ (k):

$$\tilde{BF}\,(k) = 0,\quad k = 1, 2, \ldots, N-1 \qquad (16)$$

These conditions can be fullfiled in two ways:

a) If some $\tilde{V}$ (p) or $\tilde{I}$ (q) is equal to zero (some harmonic is not present in signals), then $\tilde{H}$ (p, q) is free to select.

b) If both $\tilde{V}$ (p) and $\tilde{I}$ (q) are different from zero (since the presence of the correspondent harmonics in signals is expected), then

$$\tilde{H}\,(p,q) = 0\ .$$

If these conditions are satisfied, the (constant) value of the bilinear form will be:

$$\tilde{bf}\,(n) = \frac{1}{N}\ \tilde{BF}\,(0) = \frac{1}{N^2} \sum_{p=0}^{N-1} H\,(p,\,N-p)\,I\,(N-p) \qquad (17)$$

Taking into account that

$$\tilde{H}\,(p,\,N-p) = H^{*}(N-p,\,p) = a_p + jb_p,\quad p = 1, 2, \ldots, M \qquad (18)$$

and replacing $\tilde{V}$ (p) and $\tilde{I}$ (q) with (3) and (4), the following value of $\tilde{bf}$ (n) is obtained

$$\tilde{bf}\,(n) = a_0 V_0 I_0 + \sum_{m=0}^{M} a_m P_m + b_m Q_m$$

where P_m and Q_m are active and reactive power of the m–th harmonic.

So, the value of a constant bilinear form is a linear combination of active and reactive powers of individual harmonics.

Some examples of the possible values of a bilinear form are:

1. $a_1 = 1$, and all the other a_k and b_k are zero:
   $\tilde{bf}\,(n) = P_1$ (Active power of the first harmonic)
2. All the a_k are equal to 1, and all the b_k are equal to zero:
   $\tilde{bf}\,(n) = \sum_{k=0}^{M} P_k$ (Average power, or sum of all harmonic's active powers)
3. $b_1 = 1$, and all the other a_k and b_k are zero:
   $\tilde{bf}\,(n) = Q_1$ (Reactive power of the first harmonic)
4. All the b_k are equal to one, and all the a_k are zero:
   $\tilde{bf}\,(n) = \sum_{k=1}^{M} Q_k$ (Budeanu's definition of the reactive power)

5. All the a_k are zero, and $b_k = k$

$\widetilde{bf}(n) = \sum_{k=1}^{M} kQ_k$ (The term appears in the Kuster's definition of the inductive reactive power)

6) All the a_k are zero, and $b_k = 1/k$

$\widetilde{bf}(n) = \sum_{k=1}^{M} Q_k/k$ (The term appears in the Kuster's definition of the capacitive reactive power)

The above examples show how the value of the bilinear form can be adapted to suit many existing definitions of active and reactive power. It may be noted that since now, no digital algorithm were proposed for calculation of Budenau's and Kuster's definitions of reactive power.

The design of an algorithm based on a bilinear form follows three main steps:

I. Choose apropriate a_k and b_k.

Put $\widetilde{H}(p, N-p) \neq a_p + j\, b_p$, $p = 0, 1, \ldots, M$

$H(N-p, p) \neq a_p - j\, b_p$, $p = 1, \ldots, M$

II. The other values of $\widetilde{H}(p, q)$ are or zero, or arbitrary:

a) If $p = 0, 1, 2, M, N-M, \ldots, N-1$ or

$q = 0, 1, 2, M, N-M, \ldots, N-1$ $\widetilde{H}(p, q)$ is arbitrary but restrictions (8) must be observed.

b) All the other values of $\widetilde{H}(p, q)$ must be zero.

III. Find h_{km} as inverse DFS of $\widetilde{H}(p, q)$

$$h_{km} = \frac{1}{N^2} \sum_{p=0}^{N-1} \sum_{q=0}^{N-1} \widetilde{H}(p, q)\, W_N^{-kp}\ W_N^{-mq}$$

Two examples of algorithm design

The aim of the presentation of these examples is to demonstrate the ease of the design in bilinear form.

The first example will show that algorithm for calculation of active power of the first harmonic based on Fourier technique is a special case of a bilinear algorithm.

The second example will produce an algorithm for calculation of Budeanu's definition of reactive power. As far as the authors know this is the first algorithm in digital or analog form that solves the task.

1st Example

The considered signals have three significant harmonics (M=3) and DC part. The number of the samples in period must satisfy the nonequality derived from the Niquist rate:

$$N \geq 2M + 2 = 8 \qquad (19)$$

The lowest value of N = 8 is selected for this algorithm. The following steps determine sequence h_{km} :

I. Since $\widetilde{bf}(n) = P_1$, it follows: $a_1 = 1$, $a_0 = a_2 = a_3 = b_1 = b_2 = b_3 = 0$

$H(1, 7) = a_1 + b_1 = 1$, $\quad H(7, 1) = a_1 - jb_1 = 1$

$H(2, 6) = a_2 + jb_2 = 0$, $\quad H(6, 2) = a_2 - jb_2 = 0$

$H(3, 5) = a_3 + jb_3 = 0$, $\quad H(5, 3) = a_3 - jb_3 = 0$

$H(0, 0) = a_0 = 0$.

II. Since the fourth harmonic is absent, the elements $H(p, 4)$, $H(4, q)$ are free to select, but restrictions (8) must be respected.

All the other values of $\widetilde{H}(p, q)$ must be zero.

If the condition that $h_{km} = h_{mk}$ is added, the Table I shows values of $\widetilde{H}(p, q)$:

Table I.

p \ q	0	1	2	3	4	5	6	7
0	0	0	0	0	n_0	0	0	0
1	0	0	0	0	n_1	0	0	1
2	0	0	0	0	n_2	0	0	0
3	0	0	0	0	n_3	0	0	0
4	n_0	n_1	n_2	n_3	n_4	n_3	n_2	n_1
5	0	0	0	0	n_3	0	0	0
6	0	0	0	0	n_2	0	0	0
7	0	1	0	0	n_1	0	0	0

n_k are undetermined real coefficients.

III. The sequence h_{km} is determined as an inverse two-dimensional DFT:

$$h_{km} = \frac{1}{64} \sum_{p=0}^{7} \sum_{q=0}^{7} \widetilde{H}(p, q)\, W_8^{-kp}\ W_8^{-mq} =$$

$$= \frac{1}{32} \cos(k-m) \frac{2\pi}{8} + \varphi(k)(-1)^m + \varphi(m)(-1)^k +$$

$$+ \frac{n_4}{64} (-1)^{k+m}$$

Here $\varphi(k) = \frac{1}{32}\left(n_0 + n_1 \cos\frac{\pi k}{4} + n_2 \cos\frac{2\pi k}{4} + n_3 \cos\frac{3\pi k}{4}\right)$

If $n_0 = n_1 = n_2 = n_3 = n_4 = 0$, then h_{km} is the same as for the Fourier based algorithm.

2st Example

The signal is the same as in the previous example. The value of the bilinear form should be equal to the sum of the reactive powers of harmonics.

I. $a_0 = a_1 = a_2 = a_3 = 0$

$b_1 = b_2 = b_3 = 1$

$\widetilde{H}(1, 7) = \widetilde{H}(2, 6) = \widetilde{H}(3, 5) = j$,

$\tilde{H}(7,1) = \tilde{H}(6,2) = \tilde{H}(5,3) = -j\ ; \quad \tilde{H}(0,0) = 0$

II. The values of $\tilde{H}(p,4)$ and $\tilde{H}(4,q)$ are free to select, bur restrictions (8) have to be respected.

All the other values of $H(p,q)$ must be zero:

Values $\tilde{H}(p,q)$ are represented at the Table II. It is assumed that all the undetermined $\tilde{H}(p,q)$ are set to zero.

Table II.

	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	j
2	0	0	0	0	0	0	j	0
3	0	0	0	0	0	j	0	0
4	0	0	0	0	0	0	0	0
5	0	0	0	−j	0	0	0	0
6	0	0	−j	0	0	0	0	0
7	0	−j	0	0	0	0	0	0

III. The sequence h_{km} is determined by the expression:

$$h_{km} = \begin{cases} \dfrac{2}{64}\, \dfrac{\sin \frac{3(m-k)\pi}{8}}{\sin \frac{(m-k)\pi}{8}}\, (-1)^{\frac{k-m+1}{2}} &, (k-m) \text{ odd} \\ 0 &, (k-m) \text{ even} \end{cases}$$

The end of the example.

Conclusion

It has been shown that a bilinear form of periodic signal samples may be used to calculate arbitrary linear combination of active and reactive powers of individual harmonics, uder the assumption that sample frequency respects Niquist rate.

By proper selection of the weight attached to the individual active and reactive power, it is possible to obtain active and reactive power of the whole signal, in accordance with various definitions. The procedure of the bilinear form synthesis, based on DFS technique is developed. Two examples of the algorithm design are given.

The first example shows that Fourier based algorithm for the calculation of the active power of the first harmonic is a special case of bilinear form algorithm that solves the same task.

The second example is the first known case of an algorithm for the calculation of the Budeanu's definition of the reactive power.

References

[1] N. L. Kusters, and W.J.M. Moore, „On the Definition of Reactive Power Under Nonsinusoidal Conditions", IEEE Trans. Power App. Syst., Vol. IM 29, pp 1845–1854, Sept./Oct. 1980.

[2] C. I. Budenau, „Puisances reactives and fictives", Instytut Romain de Energie", Bucharest, Romania 1927.

[3] S. Fryze, „Active, reactive and apparent powers in nonsinusoidal systems", (In Polish) Przegled Elektrot., No. 7., pp. 193–203, 1931.

[4] W. Shepherd, P. Zakikhani, „Suggested Definition of reactive power for nonsinusoidal systems", Proc IEEE, 1972, 119, (9), pp. 1361–1662.

[5] M. Kezunović et al. „ Digital Protective Relaying Algorithms Sensitivity Study and Evaluation", IEEE Trans on Power Delivery, Vol 3, No. 3., July 1988., pp. 912–922.

[6] B. Peruničić et al. „Bilinear Form Approach to Synthesis of a Class of Electric Circuit Digital Processing Algorithms", IEEE trans. on Circuits and Systems, Vol 35., No. 9., Sep. 1988., pp. 1197–1200.

[7] T. Lobos, „Simplified Methods for Calculating the Impedance of Transmission Lines", Computer Elec. Eng., Vol 9., pp. 19–31, 1982.

[8] B. Peruničić, „The Measurement of the Phasor Quantities of the periodic signals", Internal report made for „Energoinvest" 1989.

[9] B. Peruničić et al. „Digital Signal Processing Algorithms for Parameter Measurement with low Sensitivity to Frequency Changes, PICA' 89 Seatle