15th UPEC

DUNDEE, G.B., APRIL, 84

DIGITAL PROCESSING ERRORS IN NUMERICAL DISTANCE PROTECTION PARAMETER ESTIMATION

M. Kezunovic

Energoinvest Company, Yugoslavia

B. Perunicic

University of Sarajevo, Yugoslavia

INTRODUCTION

There are three most important steps in performing digital signal processing for the purpose of protective relaying. The first step assumes definition of a mathematical model which gives relation between the power line parameters and currents and voltages. Given the discrete values of currents and voltages, as well as the mathematical model, the second step can be performed which provides solution in the form of the line parameters. Those parameters are further used as an input to the tripping decision procedure. Finally, the third step relates to the data sampling. This sampling is performed on input quantities of currents and voltages.

This paper is concerned with digital signal processing errors related to each of the mentioned steps.

TRANSMISSION LINE MODEL

The most accurate transmission line model is given by a well known partial differential equation often called the Telegraphy Equation. However, more practical model used in the protective relaying is a differential equation model given by:

$$\sum_k a_k \frac{d^k u(t)}{dt} = \sum_k b_k \frac{d^k i(t)}{dt} \qquad (1)$$

where: $u(t)$-voltage at the begining of the line
$i(t)$-current at the begining of the line

Order of the differential equation given by expression (1) depends on the number of the PI segments used to represent the transmission line. Coefficients a_k and b_k are functions of the distributed parameters, the length of the segments denoted Δx, and the number of segments. This means that the coefficients depend on the length of the line.

Very common approach is to further simplify expression (1) by approximating the transmission line with only one segment and by neglecting parameters c (capacitance) and g (conductance). In this case the following equation is obtained:

$$u(t) = (R_o+R_f)i(t)+L_o\frac{di(t)}{dt} \qquad (2)$$

where: $R_o = \rho x$
$L_o = lx$
ρ - line resistance
R_f - fault resistance
l - line inductance
x - line length

Equation (2) is further used to calculate parameters R_f and x.

Now, the first error analysis discussed in this paper is related to the validity of assumptions that hold for equation (2). It can be shown that, for the long transmission lines, existance of the parallel capacitances makes the time response of the line to appear to be longer than if there were only the inductances considered. This further means that an accurate representation of a long transmission line using only one segment causes a virtual enlargement of the parameter l. The following analysis can be performed to prove the above statement.

If an assumption is made that the impedance of the transmission line, represented with k segments is denoted $Z_k(s)$, then recursive relations for coefficients a_k^n and b_k^n (coefficients of the polinomials in the nominator and denominator of $Z_k(s)$ which are at the same time coefficients in eq(1)) can be calculated. Those relations can be further used to derive a closed form of coefficients a_o^k, b_o^k and b_1^k. If it is taken that $k\to\infty$ for a fixed length of a line, then the following coefficients of the infinite order differential equation can be derived:

$$b_o^\infty = R_f G_o \frac{sh\sqrt{R_o G_o}}{\sqrt{R_o G_o}} + ch\sqrt{R_o G_o}$$

$$a_o^\infty = R_f ch\sqrt{R_o G_o} + R_o \frac{sh\sqrt{R_o G_o}}{\sqrt{R_o G_o}} \qquad (3)$$

$$a_1^\infty = \frac{L_o}{2}\left(ch\sqrt{R_o G_o} + \frac{sh\sqrt{R_o G_o}}{\sqrt{R_o G_o}} + \frac{R_o G_o}{L_o G_o}\left[ch\sqrt{R_o G_o} - \frac{sh\sqrt{R_o G_o}}{\sqrt{R_o G_o}}\right] + \left[R_f G_o + \frac{R_o C_o R_f}{L_o}\right]\frac{sh\sqrt{R_o G_o}}{\sqrt{R_o G_o}}\right)$$

If the product $\sqrt{R_o G_o}$ is very small then the coefficients are:

$$b_o = 1$$
$$a_o = R_f + R_o \qquad (4)$$
$$a_1 = L_o + \frac{R_o^2 C_o}{3} + \frac{R_o C_o R_f}{2}$$

It is interesting to note that coefficients a_o and b_o are the same as the corresponding coefficients given in the equation (2) which is derived for one segment situation. On the other hand, value of a_1 is biger than the value of L_o which is what was stated at the begining of this discussion. It is important to note that if the transmission line is long enough, than the mentioned error present in the model given by equation (2) can be as high as 5-10% which is an error that can not be neglected.

PARAMETER ESTIMATION ERROR

In order to determine parameters of equation (2) it is needed to introduce an error function so that the following expression is obtained:

$$u(t) = (R_o + R_f)\,i(t) + L_o\frac{di(t)}{dt} + e(t) \qquad (5)$$

where e(t) stands for measurement error and model simplification.

The following discussion is related to analysis of the methods available for obtaining the equation parameters and at the same time minimizing the error function.

Most common method that can be used to solve the equation (5) is the integration method.

If it is assumed, that the moment t_k is the time when value for x is to be estimated, then by multiplying the equation (5) with a function w(t), and by integrating in the time interval $(t_k-\Delta T,\ t_k)$, the following algebraic equation can be obtained:

$$J_u = x(\rho J_i + lJ_{i'}) + R_f J_i + J_e \qquad (6)$$

where:

$$J_u = \int_{t_k-\Delta T}^{t_k} u(t)w(t)dt$$

$$J_{i'} = i(t_k)w(t_k) - i(t_k-T)w(t_k-t) - \int_{t_k-\Delta T}^{t_k} i(t)\frac{dw(t)}{dt}$$

$$J_i = \int_{t_k-\Delta T}^{t_k} i(t)w(t)dt, \quad J_e = \int_{t_k-\Delta T}^{t_k} e(t)w(t)dt$$

Now, let us select function w(t) in such a way to minimize influence of the error function e(t).

If the expression for u(t) is given by:

$$u(t) = Cw(t) + \Sigma(t) \qquad (7)$$

then it is usually required that expression:

$$\int_{t_k-\Delta T}^{t_k} \Sigma^2(t)dt \qquad (8)$$

has a minimal value and this is a condition for determination of the coefficient C. It is easy to conclude that $C=J_u$ if the following holds:

$$\int_{t_k-\Delta T}^{t_k} w^2(t)dt = 1 \qquad (9)$$

which does not violate the generality of considerations. Obviously, it is desirable that J_e is small compared to J_u, J_i and $J_{i'}$.

This is possible if $e(t)$ is orthogonal with $w(t)$ in the given time interval. Also, $w(t)$ should be similar in shape to the current, derivative of current and voltage signals. Therefore, the above conditions are required in order to minimize the influence of the error function $e(t)$ and the above discussion was aimed toward derivation of those conditions.

Further more, function $w(t)$ can also be formed by using a delayed signal of voltage or current. If the interpretation given above is used it is clear that $w(t)$ can be selected in the given way because there exists a similarity in the shape between functions $u(t)$, $i(t)$ and derivative of $i(t)$ and their delayed values. Of course it is assumed that the delay is taken in such a way to support this similarity. In this case coefficients of the algebraic equation are dermined by a correlation function.

SAMPLING RATE

The well known Nyquist sampling theorem states that a signal can be reconstructed from its samples if the time between samples is $\Delta t\pi/\omega_c$ where ω_c is the highest frequency in the signal. Equation that is used for reconstruction of the signal is given by:

$$f(t) = \sum_{k=-\infty}^{+\infty} f(k\Delta t)Sa\left[\omega_c(t-k\Delta t)\right] \qquad (10)$$

As it can be observed, in order to reconstruct a signal it is needed to have all of the signal samples from $-\infty$ to $+\infty$.

On the orther hand, in most of the digital relaying algorithms it is needed to calculate integral of the following type:

$$\int_{t_k-\Delta t}^{t_k} y(t)w(t)dt \qquad (11)$$

The question that is further considered in this section can be stated in the following way: is the sampling theorem sufficient condition which would enable calculation of the integrals given by (11) with minimal errors? Further discussion gives considerations of the sampling error which is relevant to the stated question.

If either voltage or current signal is given by $y(t)$ and if the following holds:

$$z(t)=y(t)w(t), \quad w(t)=0 \quad t\leq t_k-\Delta t,\ t>t_k; \qquad y(j\omega)=0, \quad |\omega|\geq\omega_c \qquad (12)$$

then calculation of the integral of $z(t)$ from the samples may be considered as a reconstruction of the signal $z(t)$ from it´s samples.

Fourier transform of $z(t)$ is equal to:

$$Z(j\omega) = \frac{Y(j\omega)*W(j\omega)}{2\pi} \qquad (13)$$

where: *-convolution

It can be observed that spectrum of $Z(j\omega)$ can not be bounded in the frequency domain. However, spectrum of $y(t)$ can be bounded if it passes through an antialiasing filter. Since the $w(t)$ is time bounded it can not have its frequency spectrum bounded. Therefore, convolution of the frequency bounded and frequency unbounded spectrum gives frequency unbounded spectrum. Hence, it is needed to investigate spectrum of $Z(j\omega)$ in order to find its practical width ω_p and accordingly to determine the sampling frequency which will be higher than the one determined by the Nyquist theorem.

Furthermore, it can be observed that the shape of the time bounded function $w(t)$ is important since it influences width of its frequency spectrum. It can be also seen that it is better if $w(t)$ has a gradual slopes on the sides, i.e. in points t_k and $t_k-\Delta T$.

It is particurlarly convinient if $w(t)$ is of the following shape:

$$w(t)=h(t_k-t) \qquad (14)$$

since the shape of $w(t)$ in this case is the same all the time in the window. It is also convinient since the integral in that case can be obtained by the means of digital filtering.

CONCLUSIONS

Discussions given in this paper are related to the analysis of various error sources present in the digital processing methods used for distance protection parameter estimation. It can be concluded that selection of the appropriate Transmission Line Model is very important since an inadequate choice can be a cause of significant error.

Criteria for selection of function w(t), used to multiply the model equations in order to obtain a solution using the integration method, is also a critical step as far as the error analysis is concerned. Further investigation of this problem would help better understanding and controling of yet another significant source of the digital processing errors present in numerical distance protection parameter estimation.

Finally, special attention should be given to the analysis of the sampling error since it is belived that classical interpretations of this problem are not sufficient to properly define required antialiasing filtering characteristics.

ACKNOWLEDGEMENTS

This study was supported by the funding agencies NSF from the USA and SIZ Nauke BiH from Yugoslavia through funds made available to U.S. - Yugoslav Joint Board on Scientific and Technological Cooperation.

AUTHOR'S ADDRESS

Dr. M. Kezunovic, ENERGOINVEST - IRCA
Tvornicka 3, Stup, 71000 Sarajevo,
Yugoslavia

Dr. B. Perunicic, Elektrotehnicki fakultet
Toplicka bb. Lukavica
71000 Sarajevo, Yugoslavia