MEASUREMENTS OF PHASE SHIFT USING SYNCHRONIZED SAMPLING

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ABSTRACT

Utilization of synchronized sampling for utility applications is presently being demonstrated through several pilot projects. One of them is measuring of the phase shift of two node voltages. However, changes in local frequency can cause substantial errors when a phase shift needs to be determined.

This paper presents a novel approach for the measurement of the phase shift between voltage signals at two ends of a transmission line. The technique assumes that the data is sampled at both ends at the same time. The proposed algorithm performs a direct computation of the phase shift. The algorithm is fast and accurate and it is not influenced by a possible variation of the system frequency. The algorithm may be used with an arbitrary sampling rate with no additional adjustments. Some test results obtained using computer simulations confirm the predicted algorithm properties.

INTRODUCTION

Calculation of the phases is one of the most important application of synchronized sampling [1,2]. The voltage phasors in the power network nodes are used to assess system security, predict voltage collapse, estimate power flows and warn of future system instability.

A very extensive review of the algorithms for the phasor calculations is given in [3]. All of the proposed algorithms are based on some assumptions regarding the voltage signal components. Some methods use prefiltering to obtain the fundamental component and suppress the other ones, and then proceed to find the phase. The others are based on some assumptions regarding signal content and have inherent filtering capabilities. Even if the basic signal is a pure sinusoid, the varying frequency of the signal is one of the main obstacles to the phase calculation, since the calculation requires a precise knowledge of the signal frequency.

oscillating with the frequency 2f. The oscillating term is equal to zero when the weight matrix $\{h_{km}\}$ satisfies the condition:

$$\sum_{k=0}^{N-1} \sum_{m=0}^{N-1} h_{km} \, e^{-jk\delta} \, e^{-jm\delta} = 0 \tag{4}$$

In the general case, the above condition is satisfied only for a particular value of delta. However, if the sums of antidiagonals in the weight matrix are all equal to zero, the condition (4) is satisfied for any value of delta [4]. This is a very convenient property if the frequency of the signal varies. For this reason, only such weight matrices are used for the algorithm presented in this paper.

The constant terms of bilinear and quadratic forms have the following values [5]:

$$BFHXY = \alpha(\delta) \frac{XY}{2} \cos(\psi - \phi) + \beta(\delta) \frac{XY}{2} \sin(\psi - \phi) \tag{5}$$

$$QFHX = \alpha(\delta) \cdot \frac{X^2}{2} \tag{6}$$

Coefficients $\alpha(\delta)$ and $\beta(\delta)$ are dependent on angle δ and weight matrix elements h_{km}:

$$\alpha(\delta) = \sum_{x=0}^{N-1} \sum_{m=0}^{N-1} h_{km} \cos(m-k)\delta \tag{7}$$

$$\beta(\delta) = \sum_{k=0}^{N-1} \sum_{m=0}^{N-1} h_{km} \sin(m-k)\delta \tag{8}$$

The phase shift is calculated combining bilinear and quadratic forms in the following expression:

$$\psi - \phi = \sin^{-1}\left[\frac{BFH_1XY(n)}{\sqrt{QFH_2X(n) \cdot QFH_3Y(n)}}\right] \tag{9}$$

Weight matrices H_1, H_2 and H_3 have sums of their antidiagonals equal to zero, and they also satisfy two additional conditions:

$$\alpha_1(\delta) = 0 \tag{10}$$

$$\beta_1^2(\delta) = \alpha_2(\delta) \cdot \alpha_3(\delta) \tag{11}$$

Any three matrices satisfying the above conditions may be used for the calculation of the phase shift. The proof and the particular choice of bilinear and quadratic forms are given in Appendix I.

If the value of the phase shift needs only to be compared with a threshold, then the algorithm can be simplified, and the computation burden eased, by

Table I. Test Results

Test	$A(n)$ p.u.	$B(n)$ p. u.	$f(n)$ Hz	f_s kHz	$\psi(n)^o$	Algorithm Result
1	1	1	60	12	45	45°
2	1	1	60	60	45	45°
3	1	1	60	0.240	45	45°
4	$1 + 0.1\cos(0.01\pi n)$	1	$30 + \frac{n}{40}$	12	45	See Fig. 1
5	1	1	60	12	$45 - \frac{n}{200}$	See Fig. 2a, 2b
6	1	1	60	12	$30 + \frac{3n}{10}$	See Fig. 3a, 3b

Tests #1, #2, and #3 were designed to demonstrate that the algorithm may be used for an arbitrary sampling frequency. Therefore, in these tests, only the sampling frequency was changed. The results were absolutely unaffected by the sampling frequency choice.

The algorithm is also transparent to changes in the value of amplitude and system frequency, as long as these parameters are constant in the data window. If these parameters vary in time, the expressions (5) and (6) are only approximate. But, if these changes are sufficiently slow, the error is not significant. In the test #4, the amplitude of the first signal was modulated with a subharmonic having 10% amplitude of the main signal and a frequency of 6 Hz. The frequency of both signals had a linear change of 1 Hz per cycle. Fig. 1 shows the algorithm result. Although the changes were significant from the point of view of the system, the algorithm showed quite an accurate value of the phase shift.

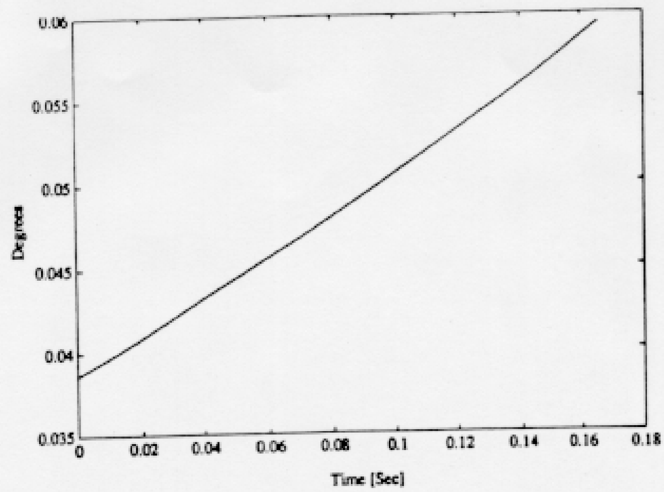

Figure 2(b): Measurement Error for Test #5

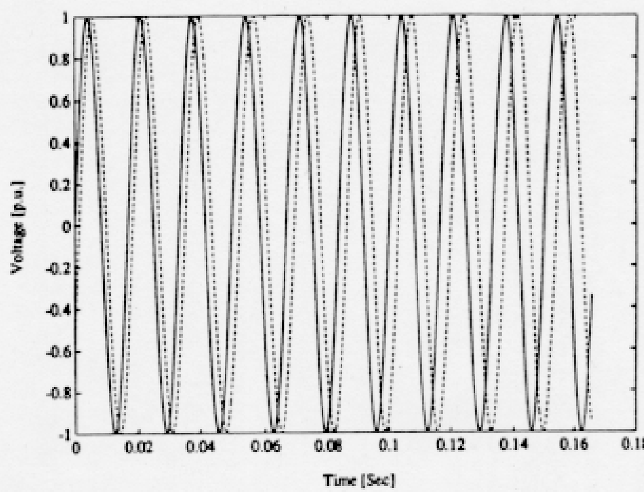

Figure 3(a): Input Signals for Test #6

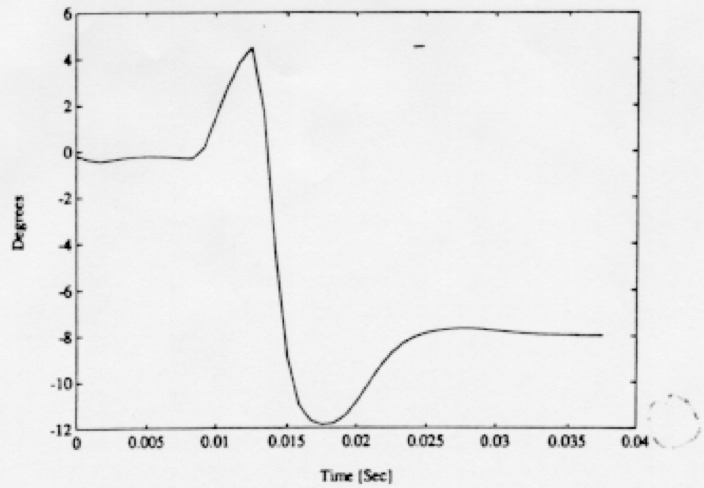

Figure 4(b): Phase Shift in Phase B Voltage for the BC Fault

CONCLUSIONS

A novel algorithm based on synchronized sampling measurement of the phase shift of two sinusoidal signals is presented. This algorithm may be implemented with an arbitrary sampling frequency without any adjustments. It is transparent to any constant system frequency and any constant value of signal amplitudes. If these parameters vary sufficiently slow, the measurement error is tolerable. The data window may be smaller than a cycle, and the calculation is performed after each new sample. This makes possible a virtually continuous measurement of the phase shift.

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REFERENCES

[1] A. G. Phadke, J. S. Thorp, "Improved Power System Protection and Control through Synchronized Phasor Measurements," Proc. of the 6th National

APPENDIX I

The bilinear form in the nominator of (9) in accordance with (5), (10) and (11) has the following value:

$$BFH_1XY(n) = \frac{XY}{2} \cdot \beta_1(\delta) \cdot \sin(\psi - \phi) \tag{A1}$$

Two quadratic forms in the denominator have the following values:

$$QFH_2X(n) = \alpha_2(\delta) \frac{X^2}{2} \tag{A2}$$

$$QFH_3Y(n) = \alpha_3(\delta) \frac{Y^2}{x} \tag{A3}$$

Since $\beta_1^2(\delta) = \alpha_2(\delta) \cdot \alpha_3(\delta)$, the value of the expression (9) is equal to:

$$\sin^{-1}\left[\frac{\frac{XY}{2} \cdot \sqrt{\alpha_2(\delta)\alpha_3(\delta)} \cdot \sin(\psi - \phi)}{\sqrt{\alpha_2(\delta) \frac{X_2}{2} \cdot \varphi_3(\delta) \cdot \frac{Y_2}{2}}}\right] = \psi - \phi \tag{A4}$$

It may be observed that all δ–dependent terms and the amplitudes cancel. That is the reason for the algorithm outcome to be transparent to both amplitudes of signals and system and sampling frequencies. Particular bilinear and quadratic forms used for these tests are:

$$BFH_1XY(n) = x_{n-3}\, y_n + 3x_{n-2}\, y_{n-1} - 3x_{n-1}\, y_{n-2} + x_n\, y_{n-3} \tag{A5}$$

$$\beta_1(\delta) = 8\sin^3\delta$$

$$QFH_2X(n) = -4x_n x_{n-2} + 4x_{n-2}^1 \tag{A6}$$

$$\alpha_2(\delta) = 8\sin^2\delta$$

$$QFH_3Y(n) = -3y_n y_{n-2} + y_n y_{n-4} + 3y_{n-1}^2 - y_{n-1} y_{n-3} \tag{A7}$$

$$\alpha_3(\delta) = 8\sin^4\delta$$

Table A–II–2. System Equivalents

Bus Name		Per-Unit Value (Ω)	Real Value (Ω)
1–3	Z_0	119.69 + j 188.93	310.248 + j 489.725
	Z_1	1.80 + j 11.44	4.666 + j 29.654
2–3	Z_0	∞	∞
	Z_1	12.58 + j 74.00	32.09 + j 191.815
1–2	Z_0	39.79 + j 100.63	103.140 + j 260.843
	Z_1	2.75 + j 18.32	7.128 + j 47.487

Table A–II–3. Self Impedances of Transmssion Lines

Bus Name		Per-Unit Value (Ω)	Real Value (Ω)
1–3	Z_0	8.94 + j 28.34	23.1734 + j 73.4601
	Z_1	1.52 + j 9.06	3.9400 + j 23.4844
1–3	Z_0	8.52 + j 29.23	22.0847 + j 75.7671
	Z_1	.38 + j 8.80	3.5771 + j 22.8105
1–3	Z_0	8.40 + j 29.37	21.7736 + j 76.1300
	Z_1	1.34 + j 8.73	3.4734 + j 22.6290
1–2	Z_1	8.42 + j 26.74	21.8255 + j 69.3128
	Z_1	1.50 + j 8.47	3.8882 + j 21.9551
2–3	Z_0	3.67 + j 12.38	9.5130 + j 32.0902
	Z_1	0.67 + j 3.92	1.7367 + j 10.1610

THE FAULT DESCRIPTION: BC FAULT AT 10% OF THE 2–3 LINE LENGTH