A NEW APPROACH TO ANALYSIS AND SYNTHESIS OF DIGITAL ALGORITHMS FOR DISTANCE RELAYING OF TRANSMISSION LINES

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Abstract. A new approach to algorithm analysis and synthesis is discussed in this paper. The new approach is based on bilinear form representation of current and voltage signal samples. It is used to study performance of digital algorithms for distance relaying of transmission lines. The non recursive algorithms are treated with the new design approach. The general design criteria derived utilizing the bilinear form approach are applied to analyze performance characteristics of a very well known Fourier transform based distance relaying algorithm. The same design criteria can be the basis of the algorithm synthesis. This is illustrated by applying the criteria in designing some new algorithms for distance relaying.

INTRODUCTION

Digital algorithms for protective relaying of transmission lines have been investigated for the last twenty years. As a result, a number of algorithms were designed and tested (Phadke and Thorp, 1988; Sachdev, 1989). Close analysis of the algorithms for distance relaying of transmission lines shows that there are at least a dozen of different algorithms suggested for this application so far. Performance evaluation of these algorithms indicates major variations in the algorithms' behavior under a reference set of fault conditions (Kezunovic and co-workers, 1988). Consequently, comparative study of distance relaying algorithms still remains as a research topic of interest. Also, investigation of new algorithms is needed since none of the existing algorithms has yet been demonstrated as having optimal performance characteristics.

The authors of this paper have introduced a new theoretical approach for analysis and synthesis of non recursive digital algorithms for power system parameter measurements (Perunicic, Kezunovic and Kreso, 1988). This new approach has been used to design new algorithms for measurement of active and reactive power as well as for frequency measurement (Kezunovic and co-workers, 1991; Perunicic, Kezunovic and Spasojevic, 1991). The same approach has been utilized to design digital algorithms for power and line parameter measurement with low sensitivity to frequency change (Perunicic and co-workers, 1990). Finally, this new approach has also been proposed for optimal design of digital distance relaying algorithms (Kezunovic, Perunicic and Levi, 1989).

This paper illustrates how the new approach can be used for both analysis of the existing algorithms as well as for design of new algorithms for distance relaying of transmission lines.

The first section of the paper outlines the algorithm design criteria derived using the new approach. Next, performance of an existing algorithm is analyzed by comparing how well it satisfies design criteria. Synthesis of new digital algorithms for distance relaying is given in the following section. Major advantages of the new approach are summarized in the conclusions.

DIGITAL ALGORITHM DESIGN CRITERIA

Bilinear Form Representation of the Algorithms

Distance relaying may be based on the algorithms derived for calculation of parameters R and L:

$$Z = R + j\omega L \tag{1}$$

The transmission line parameters R and L can be expressed as follows:

$$\begin{aligned} R &= \frac{U}{I}\cos\phi = \frac{\frac{UI}{2}\cos\phi}{\frac{I^2}{2}} = \frac{P}{(RMSI)^2} \\ \omega_0 L &= \frac{U}{I}\sin\phi = \frac{\frac{UI}{2}\cos\phi}{\frac{I^2}{2}} = \frac{Q}{(RMSI)^2} \end{aligned} \tag{2}$$

In a more general way, these expressions can be given as:

$$R = \frac{U^T C I}{I^T E I} \qquad \omega L = \frac{U^T D I}{I^T E I} \tag{3}$$

where C, D, and E are the weight matrices.

Analysis of expression (3) shows that parameters R and L can be determined using the following bilinear form:

$$BF_n = \sum_{k=0}^{N-1}\sum_{m=0}^{N-1} h_{km} u_{n-k} i_{n-m} = U^T H I \quad (4)$$

Therefore, the criteria for design of digital algorithms for R and L calculation can be derived from the condition that the bilinear forms in expression (3) have to have the corresponding values indicated by equation (2). This leads to the conditions under which the value of the bilinear form (4) becomes equal to values of P, Q and $(RMSI)^2$.

Bilinear Form Properties

Further expansion of the equation (4) indicates that for harmonic signals, the bilinear form may be expressed as a sum of a constant term FB^c and a variable term BF_n^v:

$$BF_n = BF^c + BF_n^v \quad (5)$$

These two terms can be represented as a function of the weight matrix H:

$$\begin{aligned} BF^c &= \frac{UI}{2}\left|H^c(e^{-j\psi})\right| \cdot \cos\left[\phi + \angle H^c(e^{-j\psi})\right] \\ BF_n^v &= \frac{UI}{2}\left|H^v(e^{-j\psi})\right| \cdot \cos\left[2n\psi + \phi + \angle H^v(e^{-j\psi})\right] \end{aligned} \quad (6)$$

where the related weight matrix polynomials H^c and H^v are represented as:

$$H^c(p) = \sum_{r=-N+1}^{+N-1} h_r^c \cdot p^r \quad (7)$$

with:

$$h_r^c = \sum_k \sum_m h_{km} \quad \begin{matrix} 0 \le k \le N-1 \\ k-m=r \quad 0 \le m \le N-1 \end{matrix} \quad (8)$$

and:

$$H^v(p) = \sum_{r=0}^{2N-2} h_r^v \cdot p^r \quad (9)$$

with:

$$h_r^v = \sum_k \sum_m h_{km} \quad \begin{matrix} 0 \le k \le N-1 \\ k+m=r \quad 0 \le m \le N-1 \end{matrix} \quad (10)$$

Analysis of equations (6) shows that the constant part BF^c can be used to determine active and reactive power as well as $(RMSI)^2$ values. The variable term will vanish if the following condition is fulfilled:

$$H^v(e^{-j\psi}) = 0 \quad (11)$$

This is satisfied when $e^{-j\psi}$ is a zero of the polynomial $H^v(p)$. Furthermore, the variable term will vanish for any ψ if $H^v(p)$ is identically equal to zero. This is the case when:

$$h_r^v = 0 \quad r = 0, 1, \ldots, 2N-2 \quad (12)$$

The constant part of the bilinear form is equal to active power:

$$BF^c = \frac{UI}{2}\cos\phi = P \quad (13)$$

if the polynomial $H^c(e^{-j\psi})$ is real for any value of the electrical angle ψ, and equal to 1 for a given value of the angle:

$$H^c(e^{-j\psi}) = 1, \quad \psi = \psi_0 \quad (14)$$

The constant part of the bilinear form is equal to reactive power:

$$BF^c = \frac{UI}{2}\sin\phi = Q \quad (15)$$

if the polynomial $H^c(e^{-j\psi})$ is imaginary for any value of the electrical angle ψ, and equal to $-j$ for a given value of the angle:

$$H^c(e^{-j\psi}) = -j, \quad \psi = \psi_0 \quad (16)$$

The condition for $(RMSI)^2$ calculation can be obtained from expression (6) by taking into account that both signals in the bilinear form are the current signals and that they are equal with no angle difference between them:

$$BF^c = \frac{I^2}{2} Re\left\{H^c(e^{-j\psi})\right\} \quad (17)$$

This brings the new condition:

$$Re\{H^c(e^{-j\psi})\} = 1 \quad (18)$$

which is needed to make equation (17) represent the $(RMSI)^2$ value.

Additional Design Criteria

The basic design criteria are expressed by equations (13), (15) and (17). The corresponding conditions for calculation of parameters R and L are given by equations (14), (16) and (18) respectively.

Additional conditions can be derived for design of the algorithms to make these algorithms insensitive to change of the fundamental frequency, or to make them capable of filtering out DC offset and higher harmonics.

Insensitivity to frequency change. The set of conditions for this design criteria has been discussed in an earlier reference (Perunicic and co-workers, 1990). The following is a summary of these conditions.

Design criteria for having the estimated value of parameter $\tilde{R}$ being equal to the actual value is derived from the following relation obtained by satisfying the low-sensitivity-to-frequency-change criteria:

$$\tilde{R} = \frac{Re\{C^c(e^{-j\psi})\}}{Re\{E^c(e^{-j\psi})\}} \cdot R \quad (19)$$

The estimated value $\tilde{R}$ is always equal to the actual value R, for any value of the angle ψ, if polynomials C and E are equal:

$$C = E \quad (20)$$

The following design criteria can be derived for parameter L:

$$\tilde{L} = -\frac{Im\{D^c(e^{-j\psi})\}}{Re\{E^c(e^{-j\psi})\}} \cdot \frac{\psi}{\psi_0} \cdot L \quad (21)$$

In order to make the estimated value $\tilde{L}$ to be equal to the actual value L, the following is needed:

$$-\frac{d}{d\psi} Im\{D^c(e^{-j\psi})\psi\}|_{\psi=\psi_0} = \frac{d}{d\psi} Re\{E^c(e^{-j\psi})\psi_0\}|_{\psi=\psi_0} \quad (22)$$

Obviously, for the nominal frequency, the following also holds:

$$-Im\{L^{\prime}(e^{-j\psi})\}\psi = Re\{E^{\prime}(e^{-j\psi})\}\psi_0 \qquad (23)$$

DC offset and higher harmonic filtering. The set of conditions for this design criteria has also been described in the authors' earlier reference (Kezunovic, Perunicic and Levi, 1989).

If the signal modes that need to be filtered out are specified by:

$$\begin{aligned} M &= A \cdot q^{n} \\ A &= \text{amplitude} \\ q &= e^{-\lambda\Delta t} \\ \lambda &= \text{characteristic value} \end{aligned} \qquad (24)$$

then the bilinear form value is equal to:

$$\widetilde{BF}_n = BF_n + BF_q \qquad (25)$$

A detailed analysis of the expression for BF_q indicates that the following conditions have to be met in order for BF_q to be equal to zero:

$$\begin{aligned} &\sum_{k=0}^{N-1}\sum_{m=0}^{N-1} h_{km} q^{-k} \cos m\psi_0 = 0 \\ &\sum_{k=0}^{N-1}\sum_{m=0}^{N-1} h_{km} q^{-m} \cos k\psi_0 = 0 \\ &\sum_{k=0}^{N-1}\sum_{m=0}^{N-1} h_{km} q^{-k} \sin m\psi_0 = 0 \\ &\sum_{k=0}^{N-1}\sum_{m=0}^{N-1} h_{km} q^{-m} \sin k\psi_0 = 0 \\ &\sum_{k=0}^{N-1}\sum_{m=0}^{N-1} h_{km} q^{-(k+m)} = 0 \end{aligned} \qquad (26)$$

It should be noted that the mode q will be eliminated only if the fundamental frequency is such that $\psi = \psi_0$. However, the conditions (26) indicate that mode q can be eliminated for any vlaue of ψ if the following conditions are also satisfied:

$$\begin{aligned} &\sum_{m=0}^{N-1} h_{km} q^{-m} = 0 \qquad k = 0, 1, \ldots, N-1 \\ &\sum_{m=0}^{N-1} h_{km} q^{-k} = 0 \qquad m = 0, 1, \ldots, N-1 \end{aligned} \qquad (27)$$

Therefore, this creates $2N$ additional conditions.

Presence of the Signal Noise

The approach of using bilinear forms to design algorithms for calculation of parameters R and L can also be used to design algorithms with the bias being equal to zero and the variance being minimized. The following discussion illustrates the design criteria set for the case of the input signals containing the white noise.

Let us consider the current and voltage signals in the presence of white noise:

$$\begin{aligned} \widetilde{x} &= x + e_x \\ \widetilde{y} &= y + e_y \end{aligned} \qquad (28)$$

where e_x and e_y represent stationary white noise, i.e:

$$E\{e_x\} = 0 \qquad E\{e_y\} = 0 \qquad (29)$$

The corresponding variances are:

$$E\{e_x^2\} = \sigma_x^2 \qquad E\{e_y^2\} = \sigma_y^2 \qquad (30)$$

Let us also assume that the noises of the two signals are not correlated, i.e.:

$$R_{e_x e_y}(\tau) = 0 \qquad (31)$$

In the discrete signal domain, the following is also true for the autocorrelation function:

$$R_{e_x e_x}(k-m) = \begin{cases} \sigma_{ex}^2 & k = m \\ 0 & k \neq m \end{cases} \qquad (32)$$

The main design criteria for selection of matrices of the bilinear forms can be now summarized as follows:

$$E\{\widetilde{BF}_n\} = BF_n \qquad \text{(no bias)} \qquad (33)$$

$$\sigma^2_{\widetilde{BF}_n} = \sigma^2_{min} \qquad \text{(minimal variance)} \qquad (34)$$

It is interesting to note that expression (33) is always true for the type C and D matrices found in the nominator of expression (3). For matrices of the type E, found in the denominator of expression (3), and additional condition needs to be defined based on the following equation:

$$E\left\{\widetilde{BF}_n^E\right\} = BF_n^E + \sigma^2 \sum^{N-1} k_{kk} \qquad (35)$$

This condition obviously requires that the following holds:

$$\sum_{k=0}^{N-1} h_{kk} = 0 \qquad (36)$$

ALGORITHM ANALYSIS

Bilinear Form Based Analysis

This section illustrates how the bilinear form approach can be used as an analysis tool for the algorithm performance assessment. In this case, various conditions set by the design criteria are checked to see if a given algorithm satisfies those design criteria. Since the design criteria are directly related to the performance characteristics of the algorithms, the check of the mentioned conditions gives a direct assessment of the algorithm performance.

This process is illustrated by analyzing a well known algorithm for R and L calculations. Due to the limited space, not all of the design criteria conditions are analyzed in detail. However, a detailed performance assessment can be carried out based on the set of conditions given in the previous section.

Fourier Transform Based Algorithm

Fourier transform based algorithms have been suggested for distance relaying a long time ago (Ramamoorty, 1972). Quite an efficient version of this algorithm had been developed and implemented in an actual distance relay design (Thorp and co-workers, 1979). An analysis of this algorithm is performed using the bilinear form approach. As an example of the analysis, performance assessment of the algorithm part that calculates parameter L is performed. The corresponding matrices D and E, as specified in expression (3), are determined by the algorithm design to be equal to:

$$D = \begin{bmatrix} 0 & -\sin\psi_0 & -\sin 2\psi_0 & -\sin 3\psi_0 \\ \sin\psi_0 & 0 & -\sin\psi_0 & -\sin 2\psi_0 \\ \sin 2\psi_0 & \sin\psi_0 & 0 & -\sin\psi_0 \\ \sin 3\psi_0 & \sin 2\psi_0 & \sin\psi_0 & 0 \end{bmatrix} \qquad (37)$$

$$E = \begin{bmatrix} 1 & \cos\psi_0 & \cos 2\psi_0 & \cos 3\psi_0 \\ \cos\psi_0 & 1 & \cos\psi_0 & \cos 2\psi_0 \\ \cos 2\psi_0 & \cos\psi_0 & 1 & \cos\psi_0 \\ \cos 3\psi_0 & \cos 2\psi_0 & \cos\psi_0 & 1 \end{bmatrix} \qquad (38)$$

Conditions for calculation of parameter L. An inspection of the matrices (37) and (38) shows that the following conditions are satisfied:

$$D^c(e^{-j\psi_0}) = -j \qquad (39)$$

$$E^c(e^{-j\psi_0}) = 1 \qquad (40)$$

Comparison of conditions (39) and (40) with corresponding conditions (16) and (18) indicates that these matrices indeed are providing the value of the parameter L.

Sensitivity to frequency change. To analyze algorithm sensitivity to the change in the fundamental frequency, the following expression is derived as an equivalent of condition (22):

$$\tilde{L} = \frac{2}{3}\frac{\sin 3\psi + \sqrt{2}\sin^2\psi + 3\sin\psi}{3\cos\psi - \cos 3\psi}\frac{\psi}{\psi_0} \cdot L \qquad (41)$$

For the sampling angle of $\psi_0 = 45°$, the following errors are calculated as a function of the frequency change:

ψ	42.8°	44.1°	45°	45.9°	47.2°
f%	−5	−2	0	2	5
$\tilde{L}/L$	0.9498	0.9780	1	1.0189	1.0472
% error	−5	−2.2	0	1.9	4.7

As can be observed, the algorithm is sensitive to the frequency change. Incidentialy, this was also confirmed earlier in an experimental study aimed at verifying performance of various distance relaying algorithms (Kezunovic and coworkers, 1988).

Filtering of different modes. To analyze algorithm properties regarding filtering of DC offset and higher harmonics, the following expressions are derived as an equivalent of conditions (26):

$$E^c(e^{-j\psi}) = \frac{2}{N^2}\left[\frac{\sin^2\frac{N}{4}(\psi_0-\psi)}{\sin^2\frac{\psi_0-\psi}{2}} + \frac{\sin^2\frac{N}{4}(\psi_0+\psi)}{\sin^2\frac{\psi_0+\psi}{2}}\right]$$

$$D^c(e^{-j\psi}) = -\frac{2j}{N^2}\left[\frac{\sin^2\frac{N}{4}(\psi_0-\psi)}{\sin^2\frac{\psi_0-\psi}{2}} - \frac{\sin^2\frac{N}{4}(\psi_0+\psi)}{\sin^2\frac{\psi_0+\psi}{2}}\right] \qquad (42)$$

A closer look at expression (42) shows that:

for $l = \psi = (2k+1)\psi_0$

$$E^c(e^{-jl}) = 0 \qquad D^c(e^{-jl}) = 0 \qquad (43)$$

This means, according to conditions (26), that the algorithm filters out all of the odd numbered signal harmonics.

For $l = \psi = 2k\psi_0$

$$E^c(e^{-jl}) \neq 0 \qquad D^c(e^{-jl}) \neq 0 \qquad (44)$$

This means, according to conditions (26), that the algorithm does not filter out the even harmonics and the DC offset component.

Signal noise considerations. Finally, analysis of the noise conditions, specified by equations (33) and (36), shows that the algorithm does not have the bias in the matrix D while there is a bias in the matrix E. The minimal variance, based on condition (34) is determined to be:

$$\sigma_{\tilde{L}} = \sigma_D^2 + \sigma_E^2 = \frac{1}{4}k \qquad (45)$$

where:

$$k = \frac{U^2\sigma_I^2}{2} + \frac{I^2\sigma_U^2}{2} \qquad U - RMS \text{ voltage} \quad I - RMS \text{ current}$$

ALGORITHM SYNTHESIS

Previous Results

The authors have already applied the bilinear form approach in designing new algorithms for distance relaying that are insensitive to frequency change (Perunicic and coworkers, 1988). Also, the overall methodology for synthesis of new algoirthms that are capable of filtering DC component and higher harmonics has been outlined in a previous paper (Kezunovic, Perunicic and Levi, 1989).

The following discussion explores the algorithm synthesis approach under the design criteria for signal mode filtering by giving some examples of this process.

Furthermore, this section outlines algorithm synthesis approach under the noise considerations by giving some examples of this process as well.

Basic Conditions for Synthesis

The first set of conditions is related to the selection of matrix H so that the requirement (11) is satisfied. As indicated by the expression (12), this means that the sums of the matrix elements in the anti–diagonal and all the sub–anti–diagonals have to be zero. Such matrices will be named constant–valued.

The next set of conditions is related to selection of the polynomial $H^c(e^{-j\psi})$ so that the bilinear form value gives active and reactive power as well as $(RMSI)^2$. As indicated by expressions (3), this is needed to calculate parameters R and L.

Active power calculation. It can easily be shown that the symmetric matrices defined as

$$A = A^T \qquad (46)$$

satisfy the requirement that their value is always real, i.e., that their imaginary part is always equal to zero:

$$Im\{A^c(e^{-j\psi})\} = 0, \qquad \forall\psi \qquad (47)$$

However, the symmetric matrices are not necessarily constant–valued, which is also the required condition as expressed by equation (11). One way to construct a constant-valued symmetric matrix A is to choose its elements to satisfy the following conditions:

$$\sum_k \sum_m a_{km} = -\frac{a_{rr}}{2} \qquad r = 0,1,2,\ldots N-1$$

$$0 \le k \le N-1, 0 \le m \le N-1 \quad k > m, k+m = 2r$$

$$\sum_k \sum_m a_{km} = 0 \qquad r = 0, 1, 2, \ldots N-1$$

$$0 \le k \le N-1, 0 \le m \le N-1 \quad k > m, k+m = 2r+1 \tag{48}$$

If the following condition is also satisfied:

$$Re\{A^c(e^{-j\psi})\} \neq 0, \qquad \psi = \psi_0 \tag{49}$$

then a weight matrix for real power calculation can be constructed as:

$$H_p = \frac{1}{Re\{A^c(e^{-j\psi_0})\}} \cdot A \tag{50}$$

Reactive power calculation. It can be shown that the skew-symmetric matrices defined as:

$$B^T = -B \tag{51}$$

satisfy the requirement that their value is always imaginary, i.e., that their real part is always equal to zero:

$$Re\{B^c(e^{-j\psi})\} = 0, \qquad \forall\psi \tag{52}$$

However, those matrices always satisfy the following condition as well:

$$B^v(e^{-j\psi}) = 0, \qquad \forall\psi \tag{53}$$

which is needed to make the variable part of the bilinear form to be equal to zero. In addition, if the following condition is satisfied:

$$Im\{B^c(e^{-j\psi})\} \neq 0, \qquad \psi = \psi_0 \tag{54}$$

then a weight matrix for reactive power calculation can be constructed as:

$$H_Q = \frac{1}{Im\{B^c(e^{-j\psi_0})\}} \cdot B \tag{55}$$

Algorithm insensitive to DC offset. Synthesis of such an algorithm will be illustrated by selection of the matrices required to calculate active and reactive power, which are in turn used to calculate parameters R and L.

This synthesis is made under a condition that $q = 1$ in the equation (24). In this case, the general conditions for filtering DC offset, expressed by equations (27), reduce to the following conditions:

$$\begin{aligned} \sum_{m=0}^{N-1} h_{km} = 0 \quad & k = 0, \ldots, N-1 \\ \sum_{k=0}^{N-1} h_{km} = 0 \quad & m = 0, \ldots, N-1 \end{aligned} \tag{56}$$

The conditions (56) indicate that the sum of elements in the columns and the rows of the matrix H have to be equal zero.

Active power calculation. The matrix H is to be selected to satisfy conditions (48) and (56). Let us select the following matrix:

$$H = \begin{bmatrix} 0 & 0 & -1 & -1 \\ 0 & 2 & -1 & -1 \\ -1 & -1 & 2 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix} \tag{57}$$

which satisfies the mentioned conditions. In order to normalize this matrix according to the condition (50), the value of $H^c(e^{-j\psi_0})$ is calculated according to expression (7):

$$H^c(e^{-j\psi_0}) = -2\cos 3\psi_0 - 4\cos 2\psi_0 - 2\cos\psi_0 + 4 \tag{58}$$

Finally, the matrix H_p is calculated as a ratio of expressions (57) and (58).

Reactive power calculation. The matrix H is to be selected to satisfy conditions (52), (53) and (56). Let us select the following matrix:

$$H = \begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \\ -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix} \tag{59}$$

that satisfies the mentioned conditions.

In order to normalize this matrix, according to the condition (55), the value of $H^c(e^{-j\psi_0})$ is calculated according to expression (7):

$$H^c(e^{-j\psi_0}) = -2j\sin 3\psi_0 + 4j\sin 2\psi_0 - 2j\sin\psi_0 \tag{60}$$

Again, the matrix H_Q is calculated as a ratio of expressions (59) and (60).

Noise Considerations

As can be observed from expressions (33) and (36), selection of matrices for calculation of parameters R and L without bias is straightforward. However, selection of matrices for calculation of parameters R and L, with condition (34) being satisfied, is not as obvious. Further discussion illustrates this point.

Derivation of equations for condition (34) give the following result:

$$\begin{aligned} E\{(\widetilde{BF}_n - BF_n)^2\} = {} & \sigma_y^2(X^T F X) + \sigma_x^2(Y^T L Y) \\ & + \sigma_x^2\sigma_y^2 \sum_k \sum_m h_{km}^2 \end{aligned} \tag{61}$$

where:

$$F = HH^T \qquad L = H^T H \tag{62}$$

Condition (62) also indicates that matrices F and L are always symmetric.

Furthermore, it can be shown that under the conditions set for selection of matrices for calculation of P, Q and $(RMSI)^2$ values, the following also holds:

$$F = L \tag{63}$$

Since the last term of equation (61) is smaller than the first two terms, and under the condition (63), the equation (61) becomes:

$$\sigma^2 = kF^c(e^{-j\psi_0}) + k\left|F^v(e^{-j\psi_0})\right| \cos\left[2n\psi_0 + \lfloor F^v(e^{-j\psi_0})\right] \tag{64}$$

Finally, the condition (34) reduces, based on expression (64), to the following condition:

$$\sigma_{min}^2 = kF^c(e^{-j\psi_0}) \tag{65}$$

Active power calculation. Let us select matrix H to satisfy condition (46), (48) and (49) as follows:

$$H = \begin{bmatrix} 0 & 0 & a & b \\ 0 & -2a & -b & c \\ a & -b & -2c & 0 \\ b & c & 0 & 0 \end{bmatrix} \tag{66}$$

In this case, based on equations (62), the following also holds:

$$F = L = H^2$$

In order to determine an expression for condition (65), matrix H^2 has to be calculated as follows:

$$F = H^2 = \begin{bmatrix} a^2+b^2 & b(c-a) & -2ac & 0 \\ b(c-a) & 4a^2+b^2+c^2 & 2b(a+c) & -2ac \\ -2ac & 2b(a+c) & a^2+b^2+4c^2 & b(a-c) \\ 0 & -2ac & b(a-c) & b^2+c^2 \end{bmatrix} \quad (67)$$

Finally, based on matrices H and H^2, the following expression can be calculated:

$$F^c(e^{-j\psi_0}) = -8ac\cos 2\psi_0 + 4b(a+c)\cos\psi_0 + 6a^2 + 4b^2 + 6c^2 \quad (68)$$

$$H^c(e^{-j\psi_0}) = 2b\cos 3\psi_0 + 2(a+c)\cos 2\psi_0 - 2b\cos\psi_0 - 2a - 2c \quad (69)$$

Now, the minimization process can be initiated on the expression (65) under the following additional conditions:

$$H^c(e^{-j\psi_0}) = 1 \qquad H^s(e^{-j\psi_0}) = 0 \quad (70)$$

The technique used to obtain the minimum is the Lagrangean multiplicator technique. The following function needs to be formed:

$$w(a,b,c,\lambda) = F^c(e^{-j\psi_0}) + \lambda[H^c(e^{-j\psi_0}) - 1] \quad (71)$$

For given $\psi_0 = 45°$, the following system of equations have to be solved:

$$\frac{\partial w}{\partial a} = \frac{\partial w}{\partial b} = \frac{\partial w}{\partial c} = \frac{\partial w}{\partial \lambda} = 0 \quad (72)$$

As a result, the following coefficients are obtained:

$$a = -\frac{1}{12} \quad , \quad b = -\frac{\sqrt{2}}{6} \quad , \quad c = -\frac{1}{12} \quad (73)$$

This finally gives the value for expression (65):

$$\sigma^2_{min} = 0.63k \quad (74)$$

Calculation of the minimal variance for matrices used in determining reactive power and $(RMSI)^2$ values can be carried out in the similar way as for the active power. The only difference is that the $(RMSI)^2$ case requires that the additional condition given by equation (36) is also taken into account in the minimization process expressed by conditions (72).

CONCLUSIONS

This paper illustrates how the new approach to the algorithm design can be used for both analysis and synthesis of distance relaying algorithms. The main advantage of this approach is the capability for consistent treatment of some of the most important algorithm design issues such as: insensitivity to frequency change, filtering of different signal modes, and influence of the signal noise. Further investigations of this approach are under way with a goal to design an optimal algorithm for distance relaying which would have improved performance when compared to some of the existing algorithm designs.

REFERENCES

Kezunovic, M., S. Kreso, J. T. Cain, and B. Perunicic (1988). Digital Protective Relaying Algorithm Sensitivity Study and Evaluation. IEEE TPWRD, 35, 912–921.

Kezunovic, M., B. Perunicic, and S. Levi (1989). New Methodology for Optimal Design of Digital Distance Relaying Algorithms. Proc. Intl. Conf. on Power System Protection, 235–249.

Kezunovic, M., E. Soljanin, B. Perunicic, and S. Levi (1991). New Approach to the Design of Digital Algorithms for Electric Power Measurements. IEEE TPWRD, 6, 516–523.

Perunicic, B., M. Kezunovic, and S. Kreso (1988). Bilinear Form Approach to Synthesis of a Class of Electric Circuit Digital Signal Processing Algorithms. IEEE Trans. on Circuits and Systems, 35, 1197–1200.

Perunicic, B., M. Kezunovic, S. Levi, and E. Soljanin (1990). Digital Signal Processing Algorithms for Power and Line Parameter Measurements with Low Sensitivity to Frequency Change. IEEE TPWRD, 5, 1209–1215.

Perunicic, B., M. Kezunovic, and P. Spasojevic (1991). New Digital Signal Processing Approach to the Design of the Algorithms for Frequency Deviation Measurements. Proc. IECON, 17–22.

Phadke, A. G., and J. S. Thorp (1988). Computer Relaying for Power Systems. Research Studies Press Ltd.

Ramamoorty, M., (1972). Applicatoin of Digital Computers to Power System Protection. IE(I) Journal - EL, 52, 235–238.

Sachdev, M. S. (1987). Microprocessor Relays and Protection Systems. IEEE Tutorial, 88EH0269-1-PWR.

Thorp, J. S., A. G. Phadke, S. H. Horowitz, and J. E. Beehler (1979). Limits to Impedance Relaying. IEEE Trans. on Power Apparatus and Systems, 98, 246–260.