

NEW METHODOLOGY FOR OPTIMAL DESIGN  
OF DIGITAL DISTANCE RELAYING ALGORITHMS

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ABSTRACT

This paper introduces a new methodology for optimal design of digital distance relaying algorithms. The methodology is based on bilinear forms of voltage and current samples. The new algorithm representation enables definition of the algorithms that are insensitive to the presence of the DC offset and harmonics in the signal as well as to the changes in the system frequency. Several algorithm forms are derived to illustrate how the same design methodology may be used to define different algorithms. It is shown how the new methodology may also be used to classify the existing algorithms.

## INTRODUCTION

The field of computer relaying has been developing since the late 60's. A number of digital algorithms for distance relaying were introduced up until now [1]. Each of the algorithms is developed to fit a specific set of application conditions. Several algorithms are proposed for the same application with each author making a claim that his algorithm is the "best". However, a detailed comparative study and evaluation of digital algorithms for distance relaying indicates quite different performance under specified application conditions [2].

The main problem to be resolved by digital algorithms for distance relaying is to discriminate between a no-fault and a fault situation. The decision is based on measurement of voltage and current signals. The input signals may be related to the travelling waves caused by a fault, or the input signals may be distorted fundamental frequency signals. Even though there are some papers published on the use of travelling waves for distance relaying [3], it is fair to say that the majority of the published papers are related to the algorithms based on the measurement of the distorted fundamental frequency signals [2]. This paper is concerned with the second category of the algorithms.

The main distortions in the fundamental frequency signal at the moment of the fault are related to introduction of the D.C. offset, higher harmonics and deviations in the fundamental frequency. Therefore, all of the algorithms introduced so far had to cope with the problems of accurately measuring the fundamental frequency signal under the mentioned distortions [2]. This has to be taken into account during the design phase of the relays so that different filtering and pre-processing features are used to compensate for the algorithm inaccuracies under the mentioned conditions.

This paper introduces a new algorithm design methodology which enables derivation of a generalized algorithm form. The form can be used to perform an optimal algorithm

design for a given set of input signal conditions. Therefore, this methodology can be used to design algorithms which would be insensitive to the above mentioned signal distortions.

The first part of the paper is related to the algorithm classification scheme introduced in an earlier study [4]. This classification scheme is used here to illustrate expected performance of various algorithms under different signal distortions. The second part gives the theoretical background of the Bilinear Form Approach used to derive a generalized algorithm form and the related optimal algorithm design methodology [5]. The next section illustrates the use of the new algorithm design methodology to define algorithms insensitive to the presence of D.C. offset, harmonics and fundamental frequency distortions [6]. Conclusions and Bibliography are given at the end.

## ANALYSIS OF EXISTING ALGORITHMS

### Algorithm Classification Scheme

The main algorithm classification is based on the approach used for impedance calculation [4]. As it is well known, the impedance of a transmission line may be defined in one of the two following ways:

$$Z = R + j\omega L \quad (1)$$

$$Z = \frac{V}{I} \quad (2)$$

Therefore, one class of algorithms is related to calculation of R and L based on the voltage and current measurements. In this case a differential equation model is needed as given below:

$$u(t) = Ri(t) + L \frac{di(t)}{dt} + e(t) \quad (3)$$

where:

$R, L$  – line parameters     $e(t)$  – the noise terms

An algorithm classification scheme related to the model given by equation (3) can be defined, as given in Table A in the Appendix. These algorithms are designated as Class I algorithms. The classification is based on the two basic steps needed to determine parameters  $R$  and  $L$ . One step is to treat the  $di/dt$  term, and the other step is to treat the  $e(t)$  term. The important observation is that various algorithms defined to perform these two steps may have quite different properties.

The other class of algorithms, designated as Class II, can be defined by the following model:

$$\begin{aligned}u(t) &= V \cos(\omega_0 t + \phi) + \sum_k C_k^u f_k^u + n_u(t) \\i(t) &= I \cos(\omega_0 t + \psi) + \sum_k C_k^i f_k^i + n_i(t)\end{aligned}\quad (4)$$

where:

$C_k^u, C_k^i$  – unknown coefficients

$f_k^u(t), f_k^i(t)$  – known functions representing higher harmonics and transients

$n_u(t), n_i(t)$  – noise terms

Typical estimation approach in this class usually consists of the following two steps: estimation of the first harmonic (direct and quadrature components, or amplitude and phase components); calculation of impedance as a quotient of the voltage and current phasors. This is indicated by equation (2). Again, a number of methods may be implemented to perform mentioned estimations. They primarily depend on an assumption about the complexity of the waveforms described by equation (4). Some basic methods are indicated in Table B given in the Appendix.

A detailed analysis of the algorithms that belong to the two mentioned classes reveals that the algorithm performance depends on several factors. The first group of factors are the distortions of the input waveform at the moment of the fault. The second group are the design parameters of the relaying systems. This analysis is summarized in Table I.

#### Algorithm Properties

The approach given in this paper is derived to cope primarily with the signal distortion factors. As indicated in Table I, Class I algorithms are quite insensitive to the distortion influences. However, the implementation requirements have to be carefully selected so that an optimal design is obtained. The class II algorithms are in general quite sensitive to different types of the signal distortions. On the other hand, it may be easier to select the optimal design parameters for this class. In either case it is hard to suggest what is the "best" algorithm. It is even harder to suggest what would be the methodology for selecting the "best" algorithm.

Some attempts to select the "best" algorithm [7], and to define the required methodology [8] have been proposed in the past. All of these attempts were taking into account the existing algorithm definitions. The selection process and the evaluation methodology were based on the algorithm transient testing and performance comparison. This approach has been quite useful for both the design performance evaluation [2] and for educational purposes [9]. However, the approach discussed in this paper is related to a new definition of the algorithms which enables development of a design methodology for synthesis of an optimal algorithm.

Table I. Summary of an Algorithm Property Analysis

		Class I		Class II		
		Treatment di/dt term		Treatment of the e(t) term		
		Via Samples	Via Integration	Two eg.	More eg.	
Algorithm Design Assumptions and Characteristic	Fundamental harmonic	Theoretically do not influence the results				Other comp. in the signal
	Higher harmonic and transients					Least-square
	Noise term					Linear fit
Signal Content	Analog signal Processing	Sensitive	Sensitivity may be reduced	Sensitivity is reduced	Sensitivity may be reduced	Very sens.
	Signal Measurement	Imp. resp. the same for current and voltage. Postpones the results				System function the same for $\omega_0$
Major alg. parameters	Signal sampling	Synchronization required		Synchronization is not required if sample rotation is applied		
	Data window	Optimal sampling freq. likely to exist	No conclusion about optimal sampling frequency	Optimal sampling frequency likely to exist	No conclusion about optimal sampling frequency	
Major alg. parameters	Number of Samples	Can be: fixed, selected in an optimal way, determined by the fixed number of samples and the sampling frequency, free to be selected				The same four choices
		Either depends on the sampling frequency or is not determined by any particular requirement				The same two choices

## The Generalized Algorithm Form

This section is introduced to indicate how all of the mentioned algorithms can be represented by a single generalized form. This form will serve as the basis for development of our new approach to algorithm and design methodology definition.

The class I algorithms are based on a discrete-time representation of the equation (3), with the noise term neglected:

$$J^1 u_k = R J^2 i_k + L J^3 i_k \quad (5)$$

where  $J^1, J^2, J^3$  are the operators used to convert the differential equation (3) into a difference equation. Since equation (5) is an algebraic equation with respect to  $R$  and  $L$ , two such equations defined for two different time instances are used to obtain estimates of the line parameters:

$$\begin{aligned} \tilde{R} &= \frac{J^1 u_k J^3 i_{k-1} - J^1 u_{k-1} J^3 i_k}{J^2 i_k J^3 i_{k-1} - J^2 i_{k-1} J^3 i_k} \\ \omega \tilde{L} &= \frac{J^2 i_k J^1 u_{k-1} - J^2 i_{k-1} J^1 u_k}{J^2 i_k J^3 i_{k-1} - J^2 i_{k-1} J^3 i_k} \end{aligned} \quad (6)$$

The class II algorithms are based on the voltage and current signal models given by equation (4). These models can be represented by the following general form:

$$x(t) = x_R \cdot \cos \omega_0 t + x_I \cdot \sin \omega_0 t + \sum k_{i\varphi_i}(t) \quad (7)$$

where:

$x_R, x_I$  - real and imaginary part of the fundamental frequency phasor

$k_i, \varphi_i$  - other harmonics and transients

For an assumption that the fundamental harmonic is the relevant signal model, it is possible to obtain discrete-time estimates of the voltage and current phasor components:

$$\tilde{x}_R = \alpha x_k \quad \tilde{x}_I = \beta x_k \quad (8)$$

where  $\alpha$  and  $\beta$  are operators. As a result, the line parameters can be determined using the estimated values of the phasor components:

$$\begin{aligned}\tilde{R} &= \frac{\alpha i_k \alpha u_k + \beta i_k \beta u_k}{(\alpha i_k)^2 + (\beta i_k)^2} \\ \omega \tilde{L} &= \frac{\beta u_k \alpha i_k - \alpha u_k \beta i_k}{(\alpha i_k)^2 + (\beta i_k)^2}\end{aligned}\quad (9)$$

Finally, all of the mentioned line parameter algorithms, represented by equations (6) and (9), can be defined using the following generalized form:

$$R = \frac{U^T C I}{I^T E I} \quad \omega L = \frac{U^T D I}{I^T E I} \quad (10)$$

where  $C$ ,  $D$ , and  $E$  are the weight matrices.

## BILINEAR FORM APPROACH

### General Bilinear Form Definition

As mentioned in the previous section, equations (10) are representation of the existing algorithms using the generalized form. It could be observed that both nominators and denominators of equations (10) are of the similar form which will be in our further developments designated as the bilinear form.

The general bilinear form of two sequences of samples  $x_n$  and  $y_n$  is defined by the following expression:

$$BF_n = \sum_{k=0}^{N-1} \sum_{m=0}^{N-1} h_{km} x_{n-k} y_{n-m} \quad (11)$$

where  $n$  is the discrete time when the bilinear form value is determined. The term  $h_{km}$  is a weight attached to the product of two samples  $x_{n-k}$  and  $y_{n-m}$ . The bilinear form given by equation (11) is therefore defined by the following weight matrix:

$$\mathbf{H} \hat{=} \{h_{km}\} \quad (12)$$



The matrix dimension is  $N \times N$  for the window having the width equal  $(N - 1)\Delta t$ .

Our further discussion will point out some properties of the bilinear form given by equation (11). Based on these properties, it will be shown how the value of the bilinear form can be selected to give either active power  $P$  or reactive power  $Q$ . This process is designated as a methodology for selection of properties and coefficients of the matrix  $\mathbf{H}$  so that the desired power measurements are obtained. The use of the power measurements to calculate the values of parameters  $R$  and  $L$  is explained at the end.

This final explanation concludes the definition of our methodology based on the bilinear form approach. This methodology enables one to derive different algorithms for  $R$  and  $L$  measurements.

#### Bilinear form Value for Harmonic Signals

Let us assume that the fundamental harmonic voltage and current signals are defined as:

$$\begin{aligned}u_n &= U \cos(n\psi + \phi) \\i_n &= I \cos n\psi\end{aligned}\tag{13}$$

where:

$U, I$  – signal magnitudes

$\phi$  – phase between two samples

$$\psi = \frac{2\pi\omega_o}{\omega_s} \text{ – electrical angle between two samples}$$

$\omega_o$  – system fundamental frequency

$\omega_s$  – sampling frequency

The bilinear form value for the harmonic signals defined by equation (13), at the moment  $n$ , is given as:

$$BF_n = \sum_{k=0}^{N-1} \sum_{m=0}^{N-1} h_{km} u_{n-k} i_{n-m} \quad (14)$$

$$= U^T \mathbf{H} \mathbf{I}$$

Further expansion of the equation (14) indicates that the bilinear form may be expressed as a sum of a constant term  $BF^c$  and a variable term  $BF_n^v$ :

$$BF_n = BF^c + BF_n^v \quad (15)$$

These two terms can be represented as a function of the weight matrix  $\mathbf{H}$ :

$$BF^c = \frac{UI}{2} |H^c(e^{-j\psi})| \cdot \cos[\phi + \angle H^c(e^{-j\psi})] \quad (16)$$

$$BF_n^v = \frac{UI}{2} |H^v(e^{-j\psi})| \cdot \cos[2n\psi + \phi + \angle H^v(e^{-j\psi})] \quad (17)$$

where the related weight matrix polynomials  $H^c$  and  $H^v$  are represented as:

$$H^c(p) = \sum_{r=-N+1}^{+N-1} h_r^c \cdot p^r \quad (18)$$

with:

$$h_r^c = \sum_k \sum_m h_{km} \quad \begin{array}{l} 0 \leq k \leq N-1 \\ k-m=r \quad 0 \leq m \leq N-1 \end{array} \quad (19)$$

and:

$$H^v(p) = \sum_{r=0}^{2N-2} h_r^v \cdot p^r \quad (20)$$

with:

$$h_r^v = \sum_k \sum_m h_{km} \quad \begin{array}{l} 0 \leq k \leq N-1 \\ k+m=r \quad 0 \leq m \leq N-1 \end{array} \quad (21)$$

Analysis of the equations (15), (16) and (17) indicates that the constant part  $BF^c$  can be used to determine power and line parameter values. Therefore, it is desirable to define conditions that will make the variable part  $BF_n^v$  to be equal zero. The variable term will vanish if the following condition is fulfilled:

$$H^v(e^{-j\psi}) = 0 \quad (22)$$

This is satisfied when  $e^{-j\psi}$  is a zero of the polynomial  $H^v(p)$ . Furthermore, the variable term will vanish for any  $\psi$  if  $H^v(p)$  is identically equal to zero. This is the case when:

$$h_r^v = 0 \quad r = 0, 1, \dots, 2N - 2 \quad (23)$$

#### Matrix Conditions for Different Measurements

The first set of conditions is related to the selection of matrix  $\mathbf{H}$  so that the requirement (22) is satisfied. As indicated by the expression (23), this means that the sums of the matrix elements in the anti-diagonal and all the sub-anti-diagonals have to be zero. Such matrices will be named constant-valued.

The next set of conditions is related to selection of the polynomial  $H^c(e^{-j\psi})$  so that the bilinear form value gives active and reactive power.

The active power calculation requires that the  $H^c(e^{-j\psi})$  is real for any value of the electrical angle, and equal to 1 for a given value of the angle. Therefore, if the following condition is satisfied:

$$H^c(e^{-j\psi}) = 1, \quad \psi = \psi_0 \quad (24)$$

then it can be seen from equation (16) that:

$$BF^c = \frac{UI}{2} \cos \phi = P \quad (25)$$

It can be easily shown that the symmetric matrices defined as

$$\mathbf{A} = \mathbf{A}^T \quad (26)$$

satisfy the requirement that their value is always real, i.e., that their imaginary part is always equal to zero:

$$\text{Im}\{A^c(e^{-j\psi})\} = 0, \quad \forall \psi \quad (27)$$

However, the symmetric matrices are not necessarily constant-valued, which is also the required condition as expressed by equation (22). One way to construct a constant-valued symmetric matrix  $\mathbf{A}$  is to choose its elements to satisfy the following conditions:

$$\sum_k \sum_m a_{km} = -\frac{a_{rr}}{2} \quad r = 0, 1, 2, \dots, N-1 \quad (28)$$

$$0 \leq k \leq N-1, 0 \leq m \leq N-1 \quad k > m, k+m=2r$$

$$\sum_k \sum_m a_{km} = 0 \quad r = 0, 1, 2, \dots, N-1 \quad (29)$$

$$0 \leq k \leq N-1, 0 \leq m \leq N-1 \quad k > m, k+m=2r+1$$

If the following condition is also satisfied:

$$\text{Re}\{A^c(e^{-j\psi})\} \neq 0, \quad \psi = \psi_0 \quad (30)$$

then a weight matrix for real power calculation can be constructed as

$$\mathbf{H}_p = \frac{1}{\text{Re}\{A^c(e^{-j\psi_0})\}} \cdot \mathbf{A} \quad (31)$$

The reactive power calculation requires that the polynomial  $H^c(e^{-j\psi})$  is imaginary for any value of the electrical angle, and equal to  $-j$  for a given value of the angle.

Therefore, the following condition needs to be met:

$$H^c(e^{-j\psi}) = -j, \quad \psi = \psi_0 \quad (32)$$

In this case the equation (16) gives the value of the reactive power:

$$BF^c = \frac{UI}{2} \sin \phi = Q \quad (33)$$

It can be shown that the skew-symmetric matrices defined as:

$$\mathbf{B}^T = -\mathbf{B} \quad (34)$$

satisfy the requirement that their value is always imaginary, i.e. that their real part is always equal to zero:

$$\operatorname{Re}\{B^c(e^{-j\psi})\} = 0, \quad \forall \psi \quad (35)$$

However, these matrices always satisfy the following condition as well:

$$B^v(e^{-j\psi}) = 0, \quad \forall \psi \quad (36)$$

which is needed to make the variable part of the bilinear form to be equal to zero. In addition, if the following condition is satisfied:

$$\operatorname{Im}\{B^c(e^{-j\psi})\} \neq 0, \quad \psi = \psi_0 \quad (37)$$

then a weight matrix for reactive power calculation can be constructed as:

$$\mathbf{H}_Q = \frac{1}{\operatorname{Im}\{B^c(e^{-j\psi_0})\}} \cdot \mathbf{B} \quad (38)$$

The transmission line parameter calculations are based on active and reactive power calculations. Using the following expression:

$$\begin{aligned} R &= \frac{U}{I} \cos \phi = \frac{\frac{UI}{2} \cos \phi}{\frac{P}{2}} = \frac{P}{(RMSI)^2} \\ \omega_0 L &= \frac{U}{I} \sin \phi = \frac{\frac{UI}{2} \cos \phi}{\frac{Q}{2}} = \frac{Q}{(RMSI)^2} \end{aligned} \quad (39)$$

it is possible to determine the required bilinear form matrices from the related power measurement weight matrices. The only additional value that needs to be defined is the

RMSI value. The condition for RMSI calculation can be obtained from expression (16) by taking into account that both signals in the bilinear form are the current signals and that they are equal with no angle difference between them:

$$BF^c = \frac{I^2}{2} \operatorname{Re}\{H^c(e^{-j\psi})\} \quad (40)$$

This brings the new condition:

$$\operatorname{Re}\{H^c(e^{-j\psi})\} = 1 \quad (41)$$

which is needed to make equation (40) to represent the RMSI value. This value is denoted as the quadratic form (QF):

$$QF^c = I_{rms}^2 \quad (42)$$

## APPLICATION OF THE NEW METHODOLOGY TO THE OPTIMAL ALGORITHM DESIGN

### Conditions for optimal design

The previous section has provided derivation of the matrix conditions for different measurements. These conditions have been derived for the case when the input signals are the fundamental harmonic signals only. In this case the conditions are derived to satisfy the requirement that the variable part  $BF_n^v$  is equal to zero and that the constant part  $BF^c$  has the value of the desired measurement. Therefore, the conditions for parameter R and L measurements for the input signal with no distortions are already derived in the previous section.

If a distortion in the fundamental frequency is considered, then the conditions (22), (24) and (32) are no longer satisfied. This implies that it is desirable that polynomials

$H^v$  and  $H^c$  show low sensitivity to frequency change. In order to derive the required conditions, let us represent the bilinear form expression given by equation (15), using the Taylor series expansion. This expansion is performed around the point  $\psi_0$ , which is the desired frequency out of the range of the frequency values  $\psi$ .

$$BF_n(\psi) = BF_n(\psi_0) + \frac{dBF_n(\psi)}{d\psi} \Big|_{\psi=\psi_0} \frac{\psi - \psi_0}{1!} + \frac{d^2 BF_n(\psi)}{d\psi^2} \Big|_{\psi=\psi_0} \frac{(\psi - \psi_0)^2}{2!} + \dots \quad (43)$$

If only the first two terms are considered, then the low sensitivity to frequency change translates into the condition that:

$$\frac{dBF_n(\psi)}{d\psi} = 0 \quad \text{for } \psi = \psi_0 \quad (44)$$

This further means that the following conditions are also satisfied:

$$\frac{dH^v(e^{-j\psi})}{d\psi} = 0 \quad \text{for } \psi = \psi_0 \quad (45)$$

$$\frac{dH^c(e^{-j\psi})}{d\psi} = 0 \quad \text{for } \psi = \psi_0 \quad (46)$$

If the weight matrix  $\mathbf{H}$  is selected to satisfy the condition (23), then the requirement (45) is satisfied. As far as the polynomial  $H^c$  is concerned, both real and imaginary parts have to satisfy the condition (46):

$$\frac{dRe\{H^c(e^{-j\psi})\}}{d\psi} = 0 \quad \text{for } \psi = \psi_0 \quad (47)$$

$$\frac{dIm\{H^c(e^{-j\psi})\}}{d\psi} = 0 \quad \text{for } \psi = \psi_0 \quad (48)$$

Conditions for the calculation of parameter  $R$  can be derived by using equations (10), (22), (29) and (30). As a result, the following weight matrix conditions for the estimated value  $\tilde{R}$  are obtained:

$$\tilde{R} = \frac{U^T C I}{I^T E I} \quad (49)$$

where:

$$C^v(e^{-j\psi}) = 0 \text{ for } \forall\psi$$

$$C^c(e^{-j\psi}) = \text{real number} \quad (50)$$

$$E^v(e^{-j\psi}) = 0 \text{ for } \forall\psi$$

$$E^c(e^{-j\psi}) = \text{real number} \quad (51)$$

Observing the conditions (31) and (40), it is possible to write the expression (49) in the following form:

$$\tilde{R} = \frac{\text{Re}\{C^c(e^{-j\psi})\} \cdot P}{\text{Re}\{E^c(e^{-j\psi})\} \cdot \frac{L}{2}} = \frac{\text{Re}\{C^c(e^{-j\psi})\} \cdot P}{\text{Re}\{E^c(e^{-j\psi})\} \cdot I_{rms}^2} \quad (52)$$

Taking into account equation (38), the following relation is obtained for equation (52):

$$\tilde{R} = \frac{\text{Re}\{C^c(e^{-j\psi})\}}{\text{Re}\{E^c(e^{-j\psi})\}} \cdot R \quad (53)$$

Analysis of the equation (53) suggests that the estimated value  $\tilde{R}$  is equal to the actual value R, for any value of the angle  $\psi$ , if polynomials  $C^c$  and  $E^c$  are equal:

$$C^c(e^{-j\psi}) = E^c(e^{-j\psi}), \text{ for } \forall\psi \quad (54)$$

This translates to the following condition for the corresponding weight matrices:

$$C = E \quad (55)$$

Therefore, the condition (55) needs to be satisfied in order to define an algorithm for calculation of the line parameter R so that this algorithm is insensitive to the frequency change. Obviously, in this case the condition (47) is also satisfied.

The conditions for the calculation of the parameter L can be derived in the similar manner by observing equations (9), (22), (35) and (37). In this case the estimated value



$\tilde{L}$  can be obtained as:

$$\tilde{L} \omega_0 = \frac{U^T \mathbf{D} \mathbf{I}}{I^T \mathbf{E} \mathbf{I}} \quad (56)$$

under the following conditions:

$$\begin{aligned} D^v(e^{-j\psi}) &= 0 \quad \text{for } \forall \psi \\ D^c(e^{-j\psi}) &= \text{imaginary number} \end{aligned} \quad (57)$$

$$\begin{aligned} E^v(e^{-j\psi}) &= 0 \quad \text{for } \forall \psi \\ E^c(e^{-j\psi}) &= \text{real number} \end{aligned} \quad (58)$$

Further derivation is based on the equations (38) and (40). Using these equations it is possible to write the equation (56) as:

$$\begin{aligned} \tilde{L} &= -\frac{1}{\omega_0} \frac{\text{Im}\{D^c(e^{-j\psi})\}}{\text{Re}\{E^c(e^{-j\psi})\}} \cdot \frac{Q}{\frac{I^2}{2}} \\ &= -\frac{1}{\omega_0} \frac{\text{Im}\{D^c(e^{-j\psi})\}}{\text{Re}\{E^c(e^{-j\psi})\}} \cdot \frac{Q}{I_{rms}^2} \end{aligned} \quad (59)$$

Taking into account equation (39), the following relation is obtained for equation (59):

$$\tilde{L} = -\frac{1}{\omega_0} \frac{\text{Im}\{D^c(e^{-j\psi})\}}{\text{Re}\{E^c(e^{-j\psi})\}} \cdot L \omega \quad (60)$$

Further rearrangements of equation (60) are needed:

$$\tilde{L} = -\frac{\text{Im}\{D^c(e^{-j\psi})\}}{\text{Re}\{E^c(e^{-j\psi})\}} \cdot \frac{L \omega}{\omega_0} \cdot \frac{\Delta t}{\Delta t} \quad (61)$$

to obtain the final expression as:

$$\tilde{L} = -\frac{\text{Im}\{D^c(e^{-j\psi})\}}{\text{Re}\{E^c(e^{-j\psi})\}} \cdot \frac{\psi}{\psi_0} \cdot L \quad (62)$$

In order to make the estimated value  $\tilde{L}$  to be equal to the actual value  $L$ , it is needed that:

$$-\text{Im}\{D^c(e^{-j\psi})\} \psi = \text{Re}\{E^c(e^{-j\psi})\} \psi_0 \quad (63)$$

If also the conditions (47) and (48) are needed in order to provide the low sensitivity to the frequency change, then the new condition for the equation (63) is:

$$-\frac{d}{d\psi} \operatorname{Im}\{D^c(e^{-j\psi})\psi\}|_{\psi=\psi_0} = \frac{d}{d\psi} \operatorname{Re}\{E^c(e^{-j\psi})\psi_0\}|_{\psi=\psi_0} \quad (64)$$

The consideration of the D.C. offset and higher harmonic distortions can be performed in a combined manner. Let us assume that the input signal contains, besides the fundamental harmonic, a signal mode of the following discrete form:

$$M = A \cdot q^n \quad (65)$$

where

$A$  = amplitude

$$q = e^{-\lambda \Delta t}$$

$\lambda$  = characteristic value

If the fundamental frequency signals are given by equation (13) then the distorted signal, having a mode given by equation (65) added to it, is equal to:

$$\tilde{u}_n = u_n + U_q \cdot q^n \quad (66)$$

$$\tilde{i}_n = i_n + I_q \cdot q^n$$

where  $U_q \cdot q^n$  and  $I_q \cdot q^n$  are the voltage and current modes respectively. If the distorted signal expressions (66) are substituted in the equation (14), the following expression is obtained:

$$\tilde{BF}_n = BF_n + BF_q \quad (67)$$

A detailed analysis of the expression for  $BF_q$  indicate that the following conditions have to be met in order for  $BF_q$  to be equal to zero:

$$\begin{aligned}
\sum_{k=0}^{N-1} \sum_{m=0}^{N-1} h_{km} q^{-k} \cos m\psi_0 &= 0 \\
\sum_{k=0}^{N-1} \sum_{m=0}^{N-1} h_{km} q^{-m} \cos k\psi_0 &= 0 \\
\sum_{k=0}^{N-1} \sum_{m=0}^{N-1} h_{km} q^{-k} \sin m\psi_0 &= 0 \\
\sum_{k=0}^{N-1} \sum_{m=0}^{N-1} h_{km} q^{-m} \sin k\psi_0 &= 0 \\
\sum_{k=0}^{N-1} \sum_{m=0}^{N-1} h_{km} q^{-(k+m)} &= 0
\end{aligned} \tag{68}$$

It should be noted that the mode  $q$  will be eliminated only if the fundamental frequency is such that  $\psi = \psi_0$ . However, the conditions (68) indicate that mode  $q$  can be eliminated for any value of  $\psi$  if the following conditions also are satisfied:

$$\begin{aligned}
\sum_{m=0}^{N-1} h_{km} q^{-m} &= 0 & k = 0, 1, \dots, N-1 \\
\sum_{k=0}^{N-1} h_{km} q^{-k} &= 0 & m = 0, 1, \dots, N-1
\end{aligned} \tag{69}$$

Therefore, this creates  $2N$  additional conditions.

It should be recognized that the above analysis can be applied to the D.C. component elimination if  $q = 1$ . In this case equations (13) are:

$$\tilde{u}_n = U \cos(n\psi + \phi) + U_0 \tag{70}$$

$$\tilde{i}_n = I \cos n\psi + I_0$$

In this case conditions (69) are given as:

$$\begin{aligned}
\sum_{m=0}^{N-1} h_{km} &= 0 & k = 0, 1, \dots, N-1 \\
\sum_{k=0}^{N-1} h_{km} &= 0 & k = 0, 1, \dots, N-1
\end{aligned} \tag{71}$$

The condition (71) indicates that the sums of the  $\mathbf{H}$  matrix elements in the rows and in the columns have to be equal to zero.

## Algorithm Synthesis

This section provides an illustration as how the derived methodology can be applied to the optimal algorithm design. To keep this illustration simple, only the optimal design of the algorithms for active power measurement is discussed. In the previous discussions it was shown how the power measurements are related to the parameter R and L measurements. Therefore, the following procedures can be easily applied to the synthesis of the R and L algorithms as well.

First, it will be illustrated how the basic conditions (23), (29), and (30) are used to synthesize the  $\mathbf{H}$  matrix used to calculate active power P. Let us assume that the  $\mathbf{H}$  matrix has the following form:

$$\mathbf{H}_p = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad (72)$$

Applying the mentioned conditions one obtains the following expression for the desired matrix:

$$\mathbf{H}_p = \frac{1}{2 \sin^2 \psi_0} \begin{bmatrix} 1 & -\cos \psi_0 \\ -\cos \psi_0 & 1 \end{bmatrix} \quad (73)$$

The second example is related to selection of the matrix  $\mathbf{H}$  coefficients so that insensitivity to frequency change is maintained. If again the active power measurements are of interest, then the conditions (26) (27) and (45) should be satisfied. Let us select a matrix  $\mathbf{H}$  to satisfy the conditions (26) and (27) as follows:

$$\mathbf{H}_p = \begin{bmatrix} 0 & 0 & a & b \\ 0 & -2a & -b & 0 \\ a & -b & 0 & 0 \\ b & 0 & 0 & 0 \end{bmatrix} \quad (74)$$

The condition (45) determines the values of the coefficients  $a$  and  $b$  to be equal to:

$$a = \frac{\cos 2\psi_0 + \cos^2 \psi_0}{4 \sin^4 \psi_0}, \quad b = -\frac{\cos \psi_0}{4 \sin^4 \psi_0} \quad (75)$$

Finally, an example of the synthesis of the matrix  $\mathbf{H}$  so that a mode  $q^n$  is eliminated is given. Let us select a matrix  $\mathbf{H}$  to satisfy the specific conditions (68) as well as the basic conditions (24), (28) and (29) as follows:

$$\mathbf{H}_p = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} \quad (76)$$

The general condition (69) can now be used to select the required coefficients. This procedure gives the following values for the coefficients:

$$a = \lambda \quad b = -\lambda q \quad c = \lambda q^2 \quad (77)$$

#### CONCLUSIONS

The discussion given in the paper indicates that:

- A number of digital distance relaying algorithms exist today.
- There is no straight forward methodology available to either select or synthesize an optimal algorithm.
- The methodology proposed in the paper can be used to perform both analysis of the existing algorithms and synthesis of the new ones.
- The new methodology can be used to synthesize algorithms that are insensitive to signal distortions such as change in fundamental frequency, and presence of the D.C. offset and higher harmonics.

As a final conclusion, it should be noted that the study presented in this paper is lacking consideration of the noise influence as well as the experimental results related

to the new algorithm's performance. These additional considerations have been studied and the results are being prepared for publication in a future paper.

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Table A. Algorithms of Class I

Treatment of the derivative using samples	Elimination of the derivative via integrat.	Treatment of e(t) term
$u(t_k) = Ri(t_k) + Li'(t_k) + e(t_k)$	$\int_{t_1}^{t_2} u(t) dt = R \int_{t_1}^{t_2} i(t) dt +$ $+ L [i(t_2) - i(t_1)] + \int_{t_1}^{t_2} e(t) dt.$	Two equations More than two equations
$i'(t)$ -derivative approx.	$i'(t_k) = \frac{i(t_k) - i(t_{k-1})}{\Delta t}$	$e(t) = 0$ Two algebraic equations are needed for solution of R and L:
1. $i'(t_k) = \frac{i(t_k) - i(t_{k-1})}{\Delta t}$	$\int_{t_1}^{t_2} x(t) dt = \sum_{k=n-N}^{n-1} x_k \Delta t$	$AR + BL = C$ $DR + EL = F$
2. $i'(t_k) = \frac{i(t_{k+1}) - i(t_k)}{\Delta t}$	$t_2 = n\Delta t; t_1 = (n-N)\Delta t$	$A_k R + B_k L = C_k$ $D_k R + E_k L = F_k$
3. $i'(t_k) = \frac{i(t_{k+1}) - i(t_{k-1})}{2\Delta t}$	$\int_{t_1}^{t_2} x(t) dt = \sum_{k=n-N}^{n-1} \frac{(x_k + x_{k+1})}{2} \Delta t$	by using the minimum square error criterion:
	$x_k = x(k\Delta t)$	$\frac{\partial J}{\partial R} = 0, \frac{\partial J}{\partial L} = 0, J = \int_{t_{k-T}}^{t_k} e^2(t) dt$

Table B. Algorithms of Class II

No Optimization		Optimization
Other components in the signal		
First harmonic only	Correlation	Convolution
<p>The signal models are:</p> $u(t) = V \cos(\omega_0 t + \phi)$ $i(t) = I \cos(\omega_0 t + \psi)$ <p>The signal models are plugged into the equation:</p> $u(t) = R i(t) + L \frac{di(t)}{dt}$ <p>The two equations to solve for R and L are obtained by taking two time instances <math>t_k</math> and <math>t_{k+1}</math>. The corresponding terms multiplying <math>\cos \omega_0 t</math> and <math>\sin \omega_0 t</math> from the left and right hand side are equated to obtain the two equations.</p>	<p>The signal models are:</p> $u(t) = V \cos(\omega_0 t + \phi) + \sum_k u_k F_k$ $i(t) = I \cos(\omega_0 t + \psi) + \sum_k i_k F_k$ <p>The signal is multiplied with a time dependent weight function and integrated in a "moving" window.</p> <p>The following approaches are possible:</p> <ul style="list-style-type: none"> <li>-Full Cycle Fourier Analysis</li> <li>-Half Cycle Fourier Analysis</li> <li>-Fourier Analysis with a Non-Fundamental Harmonic</li> <li>-Current and voltage are the weight functions</li> </ul>	<p>The signal models are:</p> $u(t) = V \cos(\omega_0 t + \phi) + \sum_k u_k F_k$ $i(t) = I \cos(\omega_0 t + \psi) + \sum_k i_k F_k$ <p>The following methods for optimal signal parameter estimation can be used:</p> <ul style="list-style-type: none"> <li>-Minimum least square fit</li> <li>-Linear estimation</li> <li>-Kalman filtering</li> </ul>