

# Energy Function based Approach for Online Inertia Estimation Utilizing Synchrophasor Measurements

Rakesh Kumar Panda, Abheejeet Mohapatra, S. C. Srivastava and M. Kezunovic

**Abstract**—System inertia plays a crucial role in maintaining transient stability in a Power System (PS) network during a disturbance. Increased penetration of Renewable Energy Sources (RESs) into the system, makes the system inertia a variable quantity. Hence, the estimation of the system inertia under such scenarios is becoming more pertinent. In this paper, an energy function based approach is proposed for online inertia estimation using measurements from Phasor Measurement Units (PMUs). The synchronous machine's swing equation is used to emulate the system dynamics. The Potential Energy (PE) at a bus is calculated from the PMU measurement. A novel analogy is established between the inertia constant and the rate of change of PE. Real-Time Digital Simulator (RTDS) is used to illustrate the performance of the proposed approach on the WSCC 9 bus system. The superiority of the proposed approach is investigated by comparing the results with an existing method in the literature.

**Index Terms**—Inertia Constant, Energy Function, PMU, RO-COF, Swing Equation, Synchronous Generator.

## I. INTRODUCTION

**I**NERTIA in a PS network depends on the stored Kinetic Energy (KE) in the system, and it plays a vital role in maintaining the system's angular stability during a disturbance [1]. With the growing penetration of RESs in the network, the effective system inertia reduces and becomes a variable quantity due to the intermittent generation from such sources [2]. Hence, it is vital to monitor the system inertia, to initiate necessary control action in order to improve the angular stability of the system.

In the literature, load ramp [3], probing [4], and transient tests [5] are reported for inertia estimation. In [6], an equivalent system model, which requires substantial measurement data, is used for the estimation of inertia. A switching Markov Gaussian model [7] is used to estimate the aggregated inertia using historical data measurement. A Bayesian framework for inertia estimation considering uncertainty in the system parameters is reported in [8]. These methods, in general, are computationally burdensome and may not be suitable for online transient stability assessment of the system. Further, extensive research works [6]–[12] for inertia estimation using PMU measurements are investigated in the literature. In [9], aggregated inertia is estimated using externally generated

probing signals from power electronics-based devices. This probing signal is added to the voltage reference of the automatic voltage regulator, which narrows down the viability of this approach. Inertia estimation through a step-change in load using PMU measurements is reported [10]. However, it may not be suitable for real-time applications. In [11], inertia is estimated by taking the double derivative of the voltage phase angle. An indirect adaptive control based method is used in [12] to estimate the bus inertia. This inertia estimation approaches [6]–[12], in general, have poor accuracy, dependency on the specific type of measurement, involve more computational time. Thus, there is still a need to develop a technique for online and accurate estimation of the system inertia.

Traditionally, the energy function has been used for the assessment of transient stability of the system [13]. In general, the Lyapunov theory-based direct method [14] is used for the assessment of transient stability. This method does not require obtaining closed-form solutions of the differential equations, which significantly reduces the computational time. Hence, it may be suitable for online inertia estimation. In this paper, an energy function based approach for accurate and fast estimation of inertia at a bus in the PS network is proposed. The proposed approach only requires the measurements from a PMU placed at the bus, where inertia is to be estimated. System dynamics are emulated through a swing equation. In the Lyapunov direct method [14], the energy function considers both PE and KE. However, to directly calculate KE, the system inertia has to be a known quantity. However, PE can be directly calculated from the PMU measurement data. A novel analogy is derived between the inertia constant and the rate of change of PE with time, which is further used to estimate inertia. The proposed approach is independent of the type of source, as PE is directly calculated from the PMU measurements. Results obtained using RTDS based simulation for the WSCC 9 bus system, prove the efficacy of the proposed approach over an existing method [11].

The rest of the paper is as follows. Section II discusses the procedure to calculate PE using PMU measurements. The proposed approach and steps to estimate inertia are explained in Section III. Simulated results obtained on RTDS are discussed in Section IV, followed by the conclusions in Section V.

## II. ONLINE ENERGY CALCULATION

Inertia is an essential parameter in the electrical power network and plays a vital role monitoring the system's angular stability during a disturbance. Several approaches for

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inertia enhancement, such as the approaches in [15], [16], are discussed in the literature to improve the angular stability of the system. For proper control action, the estimation of available inertia is required at the bus where controllers are connected. Consider such a bus as bus A in Fig. 1. Assume that the dynamics at bus A (due to the network), be emulated by an equivalent Synchronous Generator (SG) with an internal voltage  $E_{sg} \angle \delta_{sg}$  and transient reactance  $X'_T$ , as shown in the Fig. 1. The dynamics of the SG using the classical model can

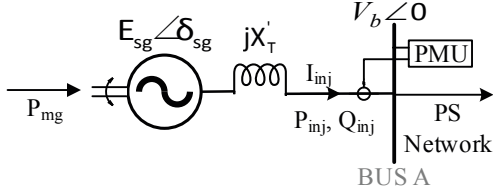


Fig. 1: Equivalent SG connected to a bus

be described by

$$\dot{\delta}_{sg} = \omega_b - \omega_0 \quad (1)$$

$$M_{sg} \frac{d^2 \delta_{sg}}{dt^2} + D_{sg} \frac{d\delta_{sg}}{dt} = P_{mg} - P_{inj} \quad (2)$$

where  $\delta_{sg}$  is the load angle,  $\omega_b$  is the actual rotor speed and  $\omega_0 = 2\pi f_0$ .  $f_0$  is the nominal system frequency,  $M_{sg}$  is the moment of inertia,  $D_{sg}$  is the damping coefficient of the equivalent SG,  $P_{mg}$  is the equivalent mechanical power input to the SG and  $P_{inj} = \frac{E_{sg} V_b}{X'_T} \sin(\delta_{sg})$  is the equivalent electrical power injected at bus A. Similarly,  $Q_{inj} = \frac{V_b^2}{X'_T} - \frac{E_{sg} V_b}{X'_T} \cos(\delta_{sg})$  is the reactive power injected at bus A. The PMU, installed at bus A, directly gives the voltage phasor, injected current phasor, frequency and the Rate of Change of Frequency (ROCOF) at the bus. Hence, real and reactive power injected at bus A can be directly evaluated as

$$P_{inj} = \sum_{k=a,b,c} V_{bk} I_{bk} \cos(\theta_{V_k} - \theta_{I_k}) \quad (3)$$

$$Q_{inj} = \sum_{k=a,b,c} V_{bk} I_{bk} \sin(\theta_{V_k} - \theta_{I_k}) \quad (4)$$

where  $.a$ ,  $.b$  and  $.c$  represent the associated quantities obtained from the PMU for phases a, b and c, respectively.

#### A. Energy Calculation at Bus A

The energy at bus A in Fig. 1 is in the form of PE and KE, which can be directly calculated using the Lyapunov energy function [13]. Some of the assumptions behind the derivation of the energy function [13], are a) Classical model of SG is used, b) governor of SG does not respond during the period of disturbance, c) damping coefficient is neglected, and d) the post-disturbance and pre-disturbance operating points are the same. Such assumptions are valid for the case of small disturbances in the system, and, thus, the energy function at bus A in Fig. 1 can be stated as

$$V_{TA} = V_{KEA} + V_{PEA} \quad (5)$$

where,  $V_{TA}$  is the total energy of the system at bus A and  $V_{KEA}$ ,  $V_{PEA}$  are the KE and the PE at bus A, respectively.

1) *KE at bus A,  $V_{KEA}$* : The KE at bus A depends on the inertia of the rotational devices connected in the system. From Fig. 1, the KE present at bus A with respect to  $\omega_0$  can be stated as

$$V_{KEA} = 0.5 M_{sg} \Delta\omega_{b0}^2 \quad (6)$$

where  $\Delta\omega_{b0} = \dot{\delta}_{sg}$ . The energy associated with the damping coefficient  $D_{sg}$  is neglected as the change in energy due to  $D_{sg}$  is always negative [13].

2) *PE at bus A,  $V_{PEA}$* : The PE at bus A can be expressed as

$$V_{PEA} = V_{PE_{mg}} + V_{PE_{X'_T}} + V_{PE_{tl}} \quad (7)$$

where  $V_{PE_{mg}} = -P_{mg}(\delta_{sg} - \delta_{sg0})$  is the PE due to the mechanical power.  $V_{PE_{X'_T}}$  and  $V_{PE_{tl}}$  are the PEs associated with the transient reactance  $X'_T$  of equivalent SG at bus A and the transmission line impedance, respectively. The expressions of  $V_{PE_{X'_T}}$  and  $V_{PE_{tl}}$  can be seen in [17]. Due to the presence of path-dependent term in the PE expression of  $V_{PE_{tl}}$ , it is difficult to calculate the PE stored in the transmission line. In [13], it has been proven that the PE stored for a lossless transmission line is equal to half of the reactive power loss. For the classical model of generator,  $V_{PE_{X'_T}}$  is equal to the half of the reactive power injected at the bus, as given in [18]. Since,  $V_{PE_{tl}}$  is negligible as compared to  $V_{PE_{X'_T}}$  [17], the combined PE can be written as

$$V_{PEA} = -P_{mg}(\delta_{sg} - \delta_{sg0}) + 0.5(Q_{inj} - Q_{inj0}) \quad (8)$$

where,  $\delta_{sg0}$  and  $Q_{inj0}$  are the steady state load angle and reactive power injection, respectively.

3) *Evaluation of PE from PMU measurement*: The PMU at a bus gives the measurements of the voltage phasor, current phasor, frequency, and ROCOF. To evaluate (8),  $P_{mg}$  and  $\delta_{sg}$  are required. However, it is difficult to get these directly by using any measurement device. To circumvent this issue and also to reduce the overall estimation time,  $P_{mg}$  is assumed to be equal to  $P_{inj0}$  due to slow governor action and also,  $\delta_{sg} \approx \theta_{Va}$  [13].  $P_{inj0}$  is the real power injected at the bus, which can be evaluated from PMU measurements by using (3) and  $\theta_{Va}$  is the associated phase angle of phase a voltage measured by PMU at bus A. Hence,  $V_{PE_{mg}} = -P_{inj0}(\theta_{Va} - \theta_{Va0})$ . Similarly from (4),  $Q_{inj}$  is directly calculated by using PMU measurements. Hence, with the help of PMU measurements, PE at bus A, i.e.  $V_{PEA}$  is evaluated.

### III. PROPOSED APPROACH FOR INERTIA ESTIMATION

The objective of the paper is to estimate inertia using PMU measurements at bus A shown in Fig. 1. Hence, the inertia constant  $M_{sg}$  in (6) is an unknown quantity.  $\Delta\omega_{b0}$  can be directly evaluated using PMU measurement. As explained in Section II-A3, PE can be calculated with the help of PMU measurements. Hence, to estimate the inertia at bus A, total energy  $V_{TA}$  in (5) has to be calculated. The total energy of

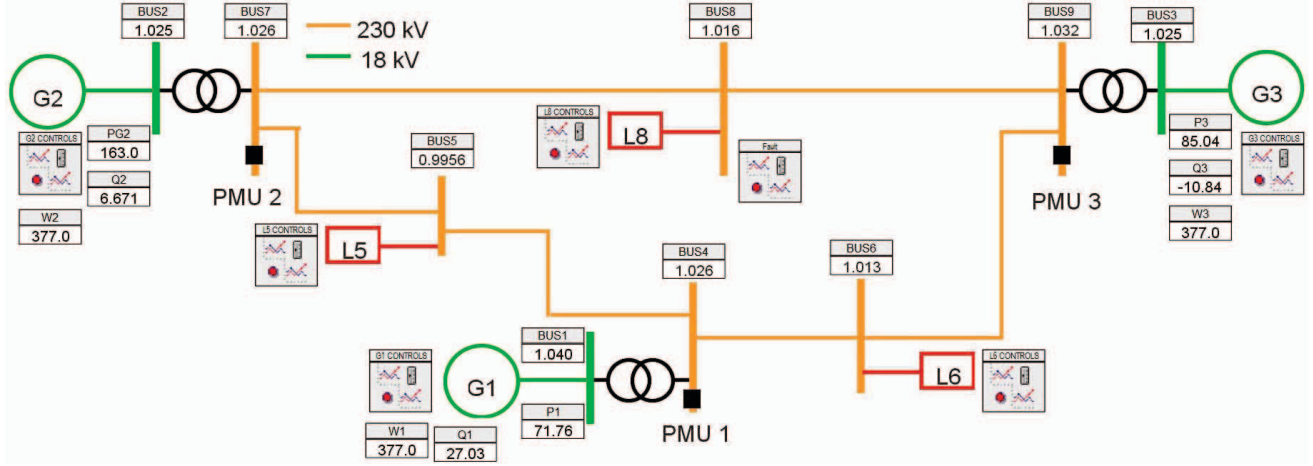


Fig. 2: One line diagram of WSCC 9 bus system

a system remains constant [13], however, it is difficult to evaluate this. The time derivative of (5) gives,

$$\dot{V}_{TA} = -D_{sg}\Delta\omega_{b0}^2 \quad (9)$$

Usually, the damping coefficient is neglected during energy function formulation, as explained in Section II-A. Hence, (9) can be rewritten as

$$\dot{V}_{TA} = 0 \Rightarrow \dot{V}_{KEA} = -\dot{V}_{PEA} \quad (10)$$

From (10), it can be concluded that by calculating the rate of change of PE  $\dot{V}_{PEA}$ , the rate of change of KE  $\dot{V}_{KEA}$  can also be evaluated. The rate of change of PE  $\dot{V}_{PEA}$  is numerically evaluated by taking the time derivative of (8), which is

$$\dot{V}_{PEA} = \frac{V_{PEA}(t) - V_{PEA}(t-1)}{\Delta t} \quad (11)$$

where  $\Delta t$  is the time gap between two consecutive samples and  $V_{PEA}(t)$  is evaluated using the corresponding PMU measurements. Similarly, the time derivative of (6) gives

$$\dot{V}_{KEA} = M_{sg}\Delta\omega_{b0}\Delta\dot{\omega}_{b0} \quad (12)$$

where  $\Delta\omega_{b0}$  and  $\Delta\dot{\omega}_{b0}$  are evaluated from PMU measurements. Thus, by using (10) and (12), inertia can be estimated as

$$M_{sg} = \frac{-\dot{V}_{PEA}}{\Delta\omega_{b0}\Delta\dot{\omega}_{b0}} \quad (13)$$

Inertia is maximum when  $\Delta\dot{\omega}_{b0}$  is maximum. Hence,  $\hat{M}_{sg}$  can be modified as

$$\hat{M}_{sg} = \frac{-\dot{V}_{PEA}}{\Delta\omega_{b0}\Delta\dot{\omega}_{b0}|_{max}} \quad (14)$$

A peak detector is used to find  $\Delta\dot{\omega}_{b0}|_{max}$ . During steady state, both  $\Delta\omega_{b0}$  and  $\Delta\dot{\omega}_{b0}$  are zero. Hence, as per (14),  $\hat{M}_{sg}$  is numerically undefined. This is due to the fact that inertia has no role, when the system is in steady state. Hence, when  $\Delta\dot{\omega}_{b0}|_{max} = 0$ , then  $\hat{M}_{sg}$  refers to the latest estimated value of inertia. It is to be noted that the proposed approach requires only PMU measurements, which does not require any further

computation. Thus, the proposed approach can be used for online inertia estimation and the associated steps are as follows

- 1) Get measurements from PMU at a bus where inertia is to be estimated.
- 2) Calculate  $Q_{inj}$  from these measurements using (4).
- 3) Evaluate  $V_{PEA}$  and  $\dot{V}_{PEA}$  using (7) and (11), respectively, from the PMU measurements (see details in Section II-A3).
- 4) Find  $\Delta\dot{\omega}_{b0}|_{max}$ .
- 5) If  $\Delta\dot{\omega}_{b0} = 0$ , then go to step 1), else estimate  $\hat{M}_{sg}$  from (14). Associated inertia constant  $\hat{H}_{sg}$  in pu is  $\pi\hat{M}_{sg}f_0$ . The percentage error in inertia estimation is  $\%Error = \frac{H_{sg} - \hat{H}_{sg}}{\hat{H}_{sg}} \times 100$ , where  $H_{sg}$  is the effective inertia of the generator(s) directly connected at the bus.

#### IV. SIMULATION RESULTS

The proposed approach for online inertia estimation is tested on the WSCC 9 bus system [19], which is shown in Fig. 2, in RTDS. Each of the three generators  $G1$ ,  $G2$  and  $G3$  are provided with a speed governor and IEEE type 1 exciter. Network base quantities are 230 kV and 100 MVA. It is assumed that PMU measurements are available at the bus, where inertia is to be estimated. However, if PMU is not available at the bus on which inertia needs to be estimated, one can utilize hybrid RTU-PMU measurement based state estimator results to estimate the inertia. In WSCC 9 bus system, inertia is estimated at bus 4, 7, and 9. Hence, one PMU is assumed to be placed at each of these buses (PMU 1 at bus 4, PMU 2 at bus 7, and PMU 3 at bus 9). Software PMU models available in RTDS has been utilized in this work. In the subsequent figures,  $PMU_{i}freq$  and  $PMU_{i}rocof$  are the frequency and ROCOF measurements from PMU  $i$ . Also,  $V_{Api} - imPU$  and  $THbjA$  are the voltage magnitude in per unit, measured by PMU  $i$  and measured voltage phase angle at  $j^{th}$  bus, respectively. Similarly,  $P_{Api} - PMU_i$  and  $Q_{Api} - PMU_i$  are  $P_{inj}$  and  $Q_{inj}$  calculated by using PMU  $i$  measurements, respectively.

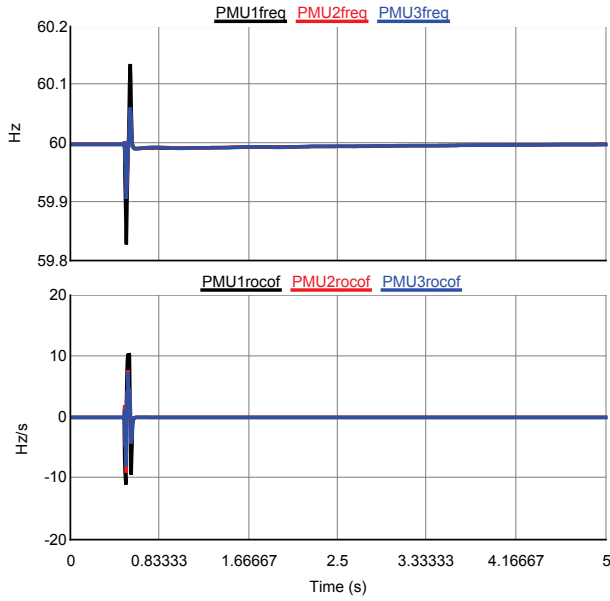


Fig. 3: Frequency and ROCOF from PMUs for load perturbation at bus 4

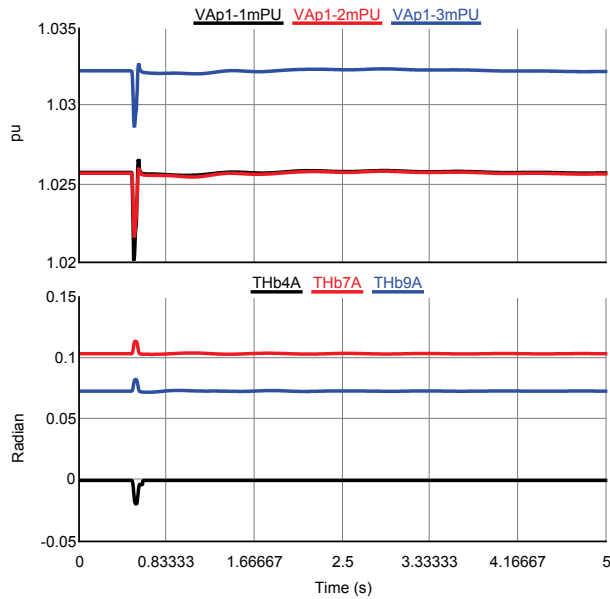


Fig. 4: Voltage phasors for load perturbation at bus 4

#### A. Inertia estimation at bus 4

To estimate the inertia at bus 4 in the WSCC 9 bus system by the proposed approach (Section III), a load perturbation of 10 MW is simulated at bus 4 for 2 cycles (0.0333 s at 60 Hz), and PMU measurements are collected at frame rate of 240 frames/s in this work. However, one can select lower frame rates dependency on the available PMUs in the system. The load perturbation initiates at 0.5 s. The frequency and ROCOF captured from the PMUs are shown in Fig. 3. It can be seen from Fig. 3 that variation in frequency and ROCOF at bus 4 is maximum due to the load perturbation at bus 4. This indicates

that for accurate inertia estimation, for which ROCOF is to be maximum, the perturbation should be simulated at the bus, where inertia is to be estimated. The voltage phasors from all PMUs during the load perturbation at bus 4 are shown in Fig. 4. By using the PMU measurements, the real power injection  $P_{inj}$  and reactive power injection  $Q_{inj}$  at the desired buses are calculated from (3) and (4) respectively and are shown in Fig. 5. By using  $Q_{inj}$  from Fig. 5 and voltage phasor measurements from Fig. 4 at bus 4, PE at bus 4 ( $PE_{sg1}$  as  $V_{PE_4}$ ) is calculated and is shown in Fig. 6. The rate of change of PE ( $\dot{V}_{PE_4}$ , plotted as  $CPE_{sg1}$  in Fig. 7) using (11) is calculated and is shown in Fig. 6. Associated estimated inertia ( $H_{sg1}$  as  $\hat{H}_{sg}$ ) variation is also shown. The estimated inertia  $\hat{H}_{sg}$  at bus 4 from the proposed approach is 22.079 s and the corresponding error is 6.6078%. Also, from Fig. 6, it can be observed that the proposed approach instantaneously estimates the inertia at bus 4. On similar lines, the inertia is estimated at bus 7 and bus 9 in the WSCC 9 bus system and the corresponding inertia estimates are given in Table I.

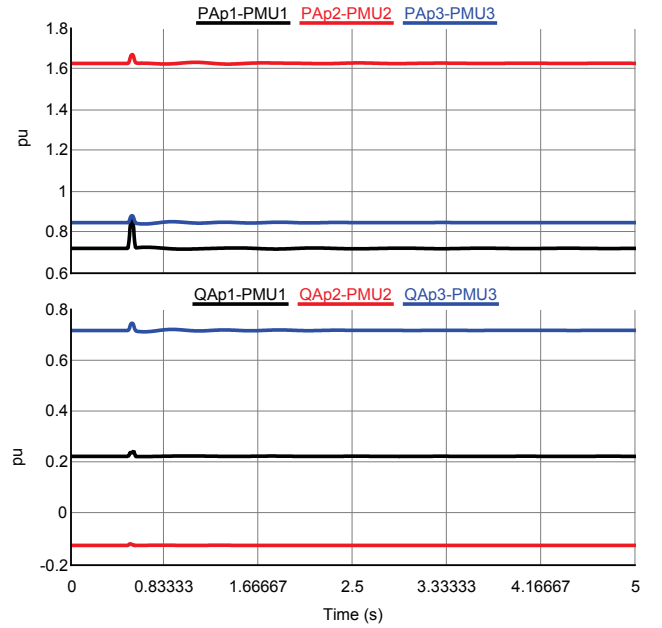


Fig. 5: Variation in  $P_{inj}, Q_{inj}$  for load perturbation at bus 4

#### B. Impact of PMU's frame rate

In order to analyze the impact of different frame rate of the PMU on the accuracy of inertia estimation using the proposed approach, inertia is individually estimated at bus 4, 7 and 9 by simulating load perturbation at each bus (on lines similar to Section IV-A) at different frame rates of the PMU, i.e. 30, 60, 120 and 240 frames/s. The associated percentage errors in estimated inertia for each bus at different PMU frame rates are shown in Fig. 7. It can be observed that the error in inertia estimation reduces as the PMU's frame rate increases and starts increasing beyond frame rate of 120 frame/s.

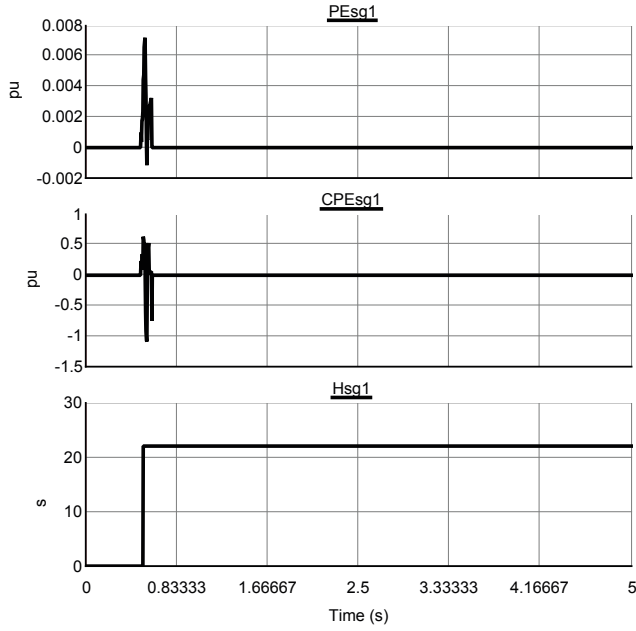


Fig. 6: Variation in  $PEsg1$  as  $V_{PE_4}$ ,  $CPEsg1$  as  $\dot{V}_{PE_4}$  and  $Hsg1$  as  $\hat{H}_{sg}$  at bus 4

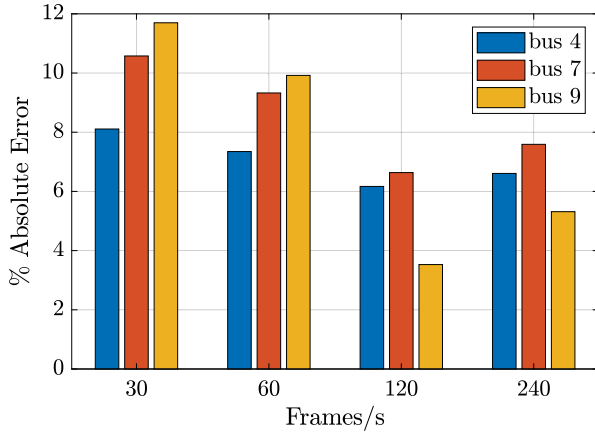


Fig. 7: Error in inertia estimation at different PMU frame rate

### C. Comparison with the approach in [11]

Table I gives the estimated inertia from proposed approach at bus 4, 7, and 9 in the WSCC 9 bus system. The associated percentage errors from the proposed approach and the approach in [11] are also given. Frame rate of PMU for inertia estimation in [11] is also chosen as 240 frames/s. It can be observed from Table I that the percentage error in estimated inertia by the proposed method varies from  $-5.3156\%$  to  $7.5906\%$ , whereas the same with [11] varies from  $6.149\%$  to  $9.238\%$ . Hence, the proposed approach is more accurate as compared to the approach in [11].

## V. CONCLUSION

In this paper, an energy function based approach is proposed for online inertia estimation by utilizing only the PMU mea-

TABLE I: Estimated Inertia from proposed approach and approach in [11]

Generator bus	Rated $H_p$ (s)	Proposed Approach		[11]
		$\hat{H}_p$ (s)	% Error	% Error
4	23.64	22.0779	6.6078	9.238
7	6.4	5.9142	7.5906	6.149
9	3.01	3.17	-5.3156	7.509

surements. The Lyapunov direct method is used to evaluate the energy in the system. Potential Energy (PE) is directly calculated from the PMU measurements. The rate of change of PE is evaluated numerically. Inertia is estimated using the rate of change of PE, frequency and ROCOF obtained from the PMU. The proposed approach is tested on the WSCC 9 bus system in RTDS. The obtained results prove that the proposed approach is suitable for online inertia estimation. It is also observed that the accuracy of estimated inertia improves with increase in frame rate of PMU. A comparison of errors in the inertia estimation by the proposed approach and an existing approach in the literature, proves that the proposed approach comparatively gives more accurate inertia estimates.

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## REFERENCES

- [1] P. Kundur, N. J. Balu, and M. G. Lauby, *Power system stability and control*. McGraw-hill New York, 1994, vol. 7.
- [2] P. Du and J. Matevosyan, "Forecast system inertia condition and its impact to integrate more renewables," *IEEE Trans. Smart Grid*, vol. 9, no. 2, pp. 1531 – 1533, Mar. 2018.
- [3] "IEEE guide: Test procedures for synchronous machines," *IEEE Std 115-1983*, pp. 1 – 90, Sep. 1983.
- [4] M. Karrari and O. P. Malik, "Identification of physical parameters of a synchronous generator from online measurements," *IEEE Trans. Energy Convers.*, vol. 19, no. 2, pp. 407 – 415, Jun. 2004.
- [5] M. A. M. Ariff, B. C. Pal, and A. K. Singh, "Estimating dynamic model parameters for adaptive protection and control in power system," *IEEE Trans. Power Syst.*, vol. 30, no. 2, pp. 829 – 839, Mar. 2015.
- [6] Z. Huang, P. Du, D. Kosterev, and S. Yang, "Generator dynamic model validation and parameter calibration using phasor measurements at the point of connection," *IEEE Trans. Power Syst.*, vol. 28, no. 2, pp. 1939 – 1949, May 2013.
- [7] X. Cao, B. Stephen, I. F. Abdulhadi, C. D. Booth, and G. M. Burt, "Switching Markov Gaussian models for dynamic power system inertia estimation," *IEEE Trans. Power Syst.*, vol. 31, no. 5, pp. 3394 – 3403, Sep. 2016.
- [8] N. Petra, C. G. Petra, Z. Zhang, E. M. Constantinescu, and M. Anitescu, "A Bayesian approach for parameter estimation with uncertainty for dynamic power systems," *IEEE Trans. Power Syst.*, vol. 32, no. 4, pp. 2735 – 2743, Jul. 2017.
- [9] J. Zhang and H. Xu, "Online identification of power system equivalent inertia constant," *IEEE Trans. Ind. Electron.*, vol. 64, no. 10, pp. 8098 – 8107, Oct. 2017.
- [10] M. Sun, Y. Feng, P. Wall, S. Azizi, J. Yu, and V. Terzija, "On-line power system inertia calculation using wide area measurements," *Int. J. Electr. Power Energy Syst.*, vol. 109, pp. 325 – 331, 2019.

- [11] Y. Zhang, J. Bank, E. Muljadi, Y. Wan, and D. Corbus, "Angle instability detection in power systems with high-wind penetration using synchrophasor measurements," *IEEE Trans. Emerg. Sel. Topics Power Electron.*, vol. 1, no. 4, pp. 306 – 314, Dec. 2013.
- [12] R. K. Panda, A. Mohapatra, and S. C. Srivastava, "Application of indirect adaptive control philosophy for inertia estimation," in *IEEE PES GTD Grand Int. Conf. Expo. Asia (GTD Asia)*, Mar. 2019, pp. 478–483.
- [13] K. Padiyar, *Structure preserving energy functions in power systems: theory and applications*. CRC Press, 2016.
- [14] N. Kakimoto, Y. Ohnogi, H. Matsuda, and H. Shibuya, "Transient stability analysis of large-scale power system by lyapunov's direct method," *IEEE Trans. Power App. Syst.*, vol. PAS-103, no. 1, pp. 160–167, Jan 1984.
- [15] J. Liu, Y. Miura, and T. Ise, "Comparison of dynamic characteristics between virtual synchronous generator and droop control in inverter-based distributed generators," *IEEE Trans. Power Electron.*, vol. 31, no. 5, pp. 3600–3611, 2015.
- [16] R. K. Panda, A. Mohapatra, and S. C. Srivastava, "An effective inertia control scheme for solar PV systems with conventional dq controller," in *2018 IEEE Power Energy Soc. Gen. Meeting*, Aug 2018.
- [17] M. Ribbens-Pavella and F. Evans, "Direct methods for studying dynamics of large-scale electric power systems—a survey," *Automatica*, vol. 21, no. 1, pp. 1 – 21, 1985.
- [18] N. Tsolas, A. Arapostathis, and P. Varaiya, "A structure preserving energy function for power system transient stability analysis," *IEEE Trans. Circuits Syst.*, vol. 32, no. 10, pp. 1041–1049, October 1985.
- [19] [Online]. Available: <https://icseg.iti.illinois.edu/wsc-9-bus-system/>