

# Optimization of Electric Vehicle Movement for Efficient Energy Consumption

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**Abstract**—This paper focuses on the development of computational optimization algorithm to provide energy-based driving guidance for electric vehicle (EV) drivers to use the limited resource efficiently. The proposed efficiency improving strategy that minimizes total energy consumption according to various driving scenarios is demonstrated and analyzed. Several factors such as regenerative energy coefficient, stop-and-go frequency, inclination of ground, wind speed, etc. have been taken into consideration in the case studies.

**Keywords**— *electric vehicle; state of charge; optimization strategy; mathematical model, regenerative energy*

## I. NOMENCLATURE

$E$	Energy consumption
$F_D$	Aerodynamic drag
$F_F$	Rolling friction
$F_G$	Road grade force
$\rho$	Air density
$A_f$	Vehicle frontal area
$C_D$	Aerodynamic drag coefficient
$v$	Instantaneous driving speed
$v_w$	Head-wind speed
$C_r$	Constant rolling friction coefficient
$C_v$	Viscous rolling friction coefficient
$m$	Mass of the vehicle
$g$	Acceleration of gravity
$\theta$	Grade angle of the road
$\eta$	Efficiency factor

## II. INTRODUCTION

Electric vehicles (EVs) introduce environmental advantages when compared to conventional gasoline-powered vehicles. They reduce air pollutants and greenhouse gas emissions while contributing to energy security through reduction in oil imports. Market penetration of EVs is foreseen to grow significantly in the future. However, the limited amount of on-board energy storage and electric range of plug-in electrical vehicles are of great concern. Besides the battery technology improvement, there is a need for an algorithm that optimizes the vehicle movement to make the most efficient use of the energy stored in the vehicle battery.

The conventional gasoline vehicle technology has reached near saturation, the focus is now on the optimization. Various

optimization methods for driving speed on different routes have been studied. Different algorithms to optimally manage the energy flow between the fuel source and batteries have been developed for hybrid electric vehicles (HEVs) [1-8]. This raises a question whether the plug-in EVs can have a similar optimal driving strategy taking into account the impact of vehicle movement on the batteries' state of charge (SOC). SOC is defined as the percentage of the remaining charge inside the battery compared to the full charge [9]. The accurate value of SOC is one of the prerequisites to develop optimal control [10]. Although it is difficult to build accurate mathematical model due to its nonlinearity [11], the SOC control strategy for EVs has been discussed in some papers [12-18].

The driving range of a plug-in EV is highly dependent on the driving profile during a trip including battery SOC, as well as vehicle performance, power consumption of key components, drivers' behavior, etc. [19]. There are also external factors such as road conditions, road slope, speed limit, rolling resistance, and aerodynamics that affects the driving range. Papers [19-21] have developed algorithms to predict, maintain or extend the available driving range. An optimization algorithm to search for global optimum to specific objective functions, which take into account the battery autonomy, driving comfort indices and the travel time, is described in [22]. In [23-24], the driving condition has been estimated. Some approaches rely on the prior knowledge or prediction of the future driving condition, while some monitor the real time operation. Some standard or designed driving cycles for different purposes are described in [25-26]. This paper does not specify the driving cycles, but analyze different driving cases by changing the variables which enables the study to be carried in a more general sense.

This paper describes the mathematical model of EV energy consumption based on the Newton's Law and proposes an optimization algorithm to realize the energy saving under different driving profiles. It is organized as follows: section I introduces the variables that are used in the mathematical model of energy consumption described in section III. Section IV analyzes the energy models with regard to different vehicle operating scenarios. In section V, the optimization algorithm is presented and several case studies are investigated. Simulation results for various scenarios are demonstrated and summarized in section VI. Finally, in section VII, conclusion is given based on the simulation results.

### III. ENERGY CONSUMPTION MATHEMATICAL MODEL FOR ELECTRIC VEHICLE DYNAMICS

There are two steps to propose the optimization algorithm. First step is to establish the energy consumption model in terms of driving conditions, while the other is to provide the optimization approach to maximize the energy efficiency with the control of different variables.

Equations or cost functions could be formulated for optimization that may result in the minimum energy consumption rate, the longest engine and battery life of use, and the highest driving security, etc. The fundamental function is based on the efficient consumption of battery energy [26-28], although other sophisticated functions including the security, dynamics and stability could also be envisioned.

In this paper, Newton's second law is applied and the model is established based on the force analysis on the vehicle during the trip. The energy consumption is a complex function of vehicle specifications, driving profiles and road conditions, such as velocity, total mass, acceleration rate, driving distance, stop-and-go frequency, regenerative braking efficiency, wind resistance, road slope, etc. The four acting forces are the aerodynamic drag  $F_D$  caused by the wind blow against the vehicle, the rolling friction  $F_F$ , the road grade force  $F_G$ , and the propulsion force  $F$  provided by the battery electricity that conduct the time derivative of momentum in the moving direction. The mathematical model of energy consumption could be written as follows:

$$F - F_D - F_F - F_G = m \left( \frac{dv}{dt} \right) \quad (1)$$

$$E = \frac{1}{\eta} \int_0^t \left( m \left( \frac{dv}{dt} \right) + F_D + F_F + F_G \right) * v * dt \quad (2)$$

Where

$$F_D = \frac{1}{2} * \rho * A_f * C_D * (v + v_w)^2 \quad (3)$$

$$F_F = (C_r + C_v * v) * m * g * \cos \theta \quad (4)$$

$$F_G = m * g * \sin \theta \quad (5)$$

The loss of energy during the transformation process from electricity to mechanical energy is ignored here. To simplify the equations, we let

$$A = \frac{1}{2} * \rho * A_f * C_D \quad (6)$$

$$B = C_r * m * g \quad (7)$$

$$C = C_v * m * g \quad (8)$$

Thus, Equation (2) can be rewritten as

$$E = \int_0^t \left( m \left( \frac{dv}{dt} \right) + A * (v + v_w)^2 + (B + C * v) * \cos \theta + m * g * \sin \theta \right) * v * dt \quad (9)$$

We choose the parameters shown in Table I below, then A in (6) is calculated as 0.5085, B in (7) as 186.984, and C in (8) as 1.5582.

TABLE I. CHOICE OF PARAMETERS (NISSAN LEAF MODEL)

$\rho$	$A_f$	$C_D$	$C_r$	$C_v$	$v_w$	$m$	$g$	$\theta$
1.28 kg/m <sup>3</sup>	2.74 m <sup>2</sup>	0.29	0.012 N	10 <sup>-4</sup> N·s/m	0 m/s	1520+70 (driver) kg	9.8 m/s <sup>2</sup>	0

### IV. ELECTRIC VEHICLE ENERGY MODELS WITH DIFFERENT DRIVING SCENARIOS

#### A. Driving at constant velocity

Generally, for each stop-and-go phase, the EV is moving a distance of  $s$  through acceleration, constant velocity  $v$ , and deceleration phases. The energy consumption during constant velocity period can be formulated as:

$$\begin{aligned} E_2 &= \int_0^t (A * v^2 + (B + C * v)) * v * dt \\ &= \int_0^{(s - \frac{v^2}{a})/v} (A * v^3 + B * v + C * v^2) * dt \\ &= (A * v^2 + B + C * v) * \left( s - \frac{v^2}{a} \right) \end{aligned} \quad (10)$$

#### B. During acceleration period

Let the acceleration rate to be constant  $a$  and the objective speed to be  $v$ . Then the energy consumption during the acceleration period is:

$$\begin{aligned} E_1 &= \int_0^{v/a} \left( m * a + A * (a * t)^2 + (B + C * (a * t)) \right) * \\ &\quad (a * t) * dt = \frac{1}{2} m v^2 + \frac{A v^4}{4a} + \frac{B v^2}{2a} + \frac{C v^3}{3a} \end{aligned} \quad (11)$$

#### C. During deceleration/ braking period

Let the deceleration rate to be constant  $a$  and the starting speed to be  $v$ . For vehicle driving, the manpower stamping on the braking pedal is also a significant factor to be considered. Without regenerative energy, the energy consumption during deceleration period should be the kinetic energy loss, which is stored in the vehicle during accelerating,

$$E_3 = 0 \quad (12)$$

If the regenerative energy is taken into consideration, some of the kinetic energy supplied by the battery to speed up can be recovered by transforming back to the battery when braking. Let  $\xi$  be the fraction of the kinetic energy that can be stored in the battery when decelerating the vehicle. Then the energy consumption during the deceleration period is:

$$E'_3 = (E_s - E_e) * (-\xi) = \frac{1}{2} m v^2 * (-\xi) \quad (12)'$$

Where  $E_s$  and  $E_e$  are the kinetic energy at the beginning and the end of the braking process, respectively.  $E'$  denotes the energy consumption in the case of considering regenerative braking.

If  $v=45$  mile/hr and  $a=2.5$  m/s<sup>2</sup>, the ratio of the energy loss during acceleration (due to aerodynamic drag, rolling friction, road grade force, etc.) to the stored kinetic energy is

$$\frac{\frac{A v^4}{4a} + \frac{B v^2}{2a} + \frac{C v^3}{3a}}{\frac{1}{2} m v^2} \approx 0.0779 \quad (13)$$

Thus, if technology permits such that the kinetic energy can be totally restored back to the battery, the highest efficiency due to regenerative braking can be up to 92.78%.

## V. OPTIMIZATION ALGORITHM AND CASE STUDY

### A. Flowchart for the optimization algorithm

Mathematically, the optimal problem is to find the global optimal driving velocity that minimize the energy consumption of EV during the trip:

$$\text{Minimize: } \min E(v, a, n, v_w, \theta) \text{ or } E'(v, a, n, \eta, v_w, \theta) \quad (14a)$$

$$\text{Subject to: } v \in V_{limit} \quad (14b)$$

$$v \leq \min\left(\sqrt{\frac{s \cdot a}{(n+1)}}, V_{max}\right) \quad (14c)$$

$$\text{Given variables: } a, n, v_w, \theta \text{ or } a, n, \eta, v_w, \theta \quad (14d)$$

Once the vehicle specifications are fixed and the road conditions are predicted according to the driving cycles preset by the vehicle owners and the GPS data, the energy consumption becomes a nonlinear function with regard to velocity and stop frequency. The optimal velocity is restricted by the constraint of maximum velocity bound, speed limit [20] and the velocity-stop frequency relationship. The optimal speed should meet the requirement of the speed limit areas

$$V_{limit} = \{v_i \leq V_i^{max}, \text{ for } (d_i^{start}, d_i^{end}), i \in \{1, 2, \dots, n_l\}\}$$

where  $n_l$  is the number of speed limited areas,  $d_i^{start}, d_i^{end}$  are the start and the end locales for area  $i$  respectively, and  $V_i^{max}$  is the maximum allowed speed.

Flowchart of the algorithm is shown in Fig. 1.

### B. Case 1: Driving a distance without stop

As described in the previous section, the total energy consumption estimation  $E$  should be the addition of the three periods ((10)-(12))

$$E = E_1 + E_2 + E_3 \quad (15)$$

$$E' = E_1 + E_2 + E_3' \quad (15)'$$

$$E_1 + E_3 = \frac{1}{2}mv^2 + \frac{Av^4}{4a} + \frac{Bv^2}{2a} + \frac{Cv^3}{3a} \quad (16)$$

$$E_1 + E_3' = \frac{1}{2}mv^2 * (1 - \xi) + \frac{Av^4}{4a} + \frac{Bv^2}{2a} + \frac{Cv^3}{3a} \quad (16)'$$

First, since there is only one acceleration/ deceleration, we need to compare the energy consumption during accelerating /decelerating periods and the constant speed period to determine whether the former value can be disregarded. According to Nissan Leaf customer reviews [29], Nissan Leaf can drive at a speed of up to 90 mph and deliver a 0-60 mph time of around 10 seconds. Thus, in this study, let the average acceleration rate to be  $a = 2.5 \text{ m/s}^2$ . While the constant velocity is 45 mile/hr, the distance be 5000 m, and the coefficient  $\xi$  be 0.06, the estimated

$$E_2/(E_1 + E_3') \approx 6.34 \quad (17)$$

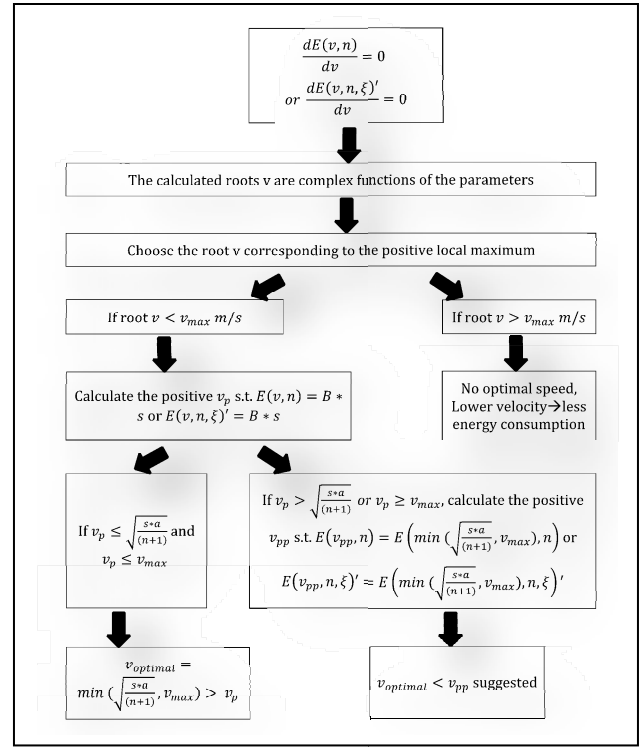


Fig. 1. Flowchart of the optimization algorithm

Therefore, for the given acceleration rate, objective speed and total distance, even when the driving cycle does not include any stop, the accelerating /decelerating periods cannot be neglected. Since driving without stop is not an exceptional case, the optimization analysis will be formulated and discussed together with case 2 with the stop times  $n=0$ .

In addition, the constraint condition for the constant speed depending on the stop times is as follows:

$$v \leq \sqrt{\frac{s \cdot a}{(n+1)}} \quad (18)$$

Thus, theoretically, for the objective speed  $v = 45$  mile/hr, the maximum stop times is 30. If the constant speed is different for each stop-and-go phase, the  $v$  in (18) will be the quadratic mean of those speed values.

### C. Case 2: Driving a distance with multiple stops

The accelerating and decelerating periods cannot be neglected for driving cycles with multiple stops. According to (15), the energy consumption including  $E_1, E_2, E_3$ , for  $n$  stops, is a function of the mean velocity as:

$$E(v, n) = (n + 1) * (E_1 + E_3) + E_2(v, n) = (n + 1) * \left( \frac{1}{2}mv^2 + \frac{Av^4}{4a} + \frac{Bv^2}{2a} + \frac{Cv^3}{3a} \right) + (A * v^2 + B + C * v) * \left( s - (n + 1) * \frac{v^2}{a} \right) \quad (19)$$

When the regenerative energy is considered, Equation (19)' is used to compute the total energy consumption:

$$E(v, n, \xi)' = (n + 1) * (E_1 + E_3') + E_2(v, n) = (n + 1) *$$

$$\left(\frac{1}{2}mv^2 * (1 - \xi) + \frac{Av^4}{4a} + \frac{Bv^2}{2a} + \frac{Cv^3}{3a}\right) + (A * v^2 + B + C * v) * \left(s - (n + 1) * \frac{v^2}{a}\right) \quad (19)'$$

To evaluate the optimal velocity with the minimum energy consumption, the derivative of the formula with respect to the velocity is analyzed.

$$\frac{dE(v,n)}{dv} = (n + 1) * \left(mv + \frac{Av^3}{a} + \frac{Bv}{a} + \frac{Cv^2}{a}\right) - \frac{4Av^3(n+1)}{a} - \frac{3Cv^2(n+1)}{a} + \left(2As - \frac{2B(n+1)}{a}\right)v + Cs = 0 \quad (20)$$

$$\frac{dE(v,n,\xi)'}{dv} = (n + 1) * \left(mv * (1 - \xi) + \frac{Av^3}{a} + \frac{Bv}{a} + \frac{Cv^2}{a}\right) - \frac{4Av^3(n+1)}{a} - \frac{3Cv^2(n+1)}{a} + \left(2As - \frac{2B(n+1)}{a}\right)v + Cs = 0 \quad (20)'$$

## VI. SIMULATION RESULT

In the case studies, let  $a = 2.5 \text{ m/s}^2$  according to [29].

### A. Case 1: Computational Results for $n=0$ :

It can be seen from the plots in Table II that, based on mathematic theories, energy consumption at  $v_2 = -1.180 \text{ m/s}$  should be a local minimum value, while the energy consumptions at  $v_1 = -104.439 \text{ m/s}$  (ignore) and  $v_3 = 103.577 \text{ m/s}$  should be the local maximum values. If the speed range of the vehicle is from 0 up to 90 mph, then the energy consumption at  $v = -1.180 \text{ m/s}$  is a global minimum which is not practical, and the value will be increasing with the increase of the driving speed through 90 mph (40 m/s). In addition, the speed constraint requires that  $v \leq 111.8 \text{ m/s}$ . The minimum energy consumption for driving this distance thus will be at least 0.2597 kWh (1.1% of SOC consumed for a 24 kWh battery) when the speed is close to zero. The analysis result can also be indicated in Fig. 3. Please note that the energy consumption value has been transformed to the unit of kWh in this paper.

It can be seen from the plots in Table III that the energy consumption at  $v_2 = -1.507 \text{ m/s}$  should be a local minimum value, while the energy consumptions at  $v_1 = -92.313 \text{ m/s}$  (ignore) and  $v_3 = 91.777 \text{ m/s}$  should be the local maximum values. If the speed range of the vehicle is from 0 up to 90 mph, then the energy consumption at  $v = -1.506 \text{ m/s}$  is a global minimum which is not practical, and the value will be increasing with the increase of the driving speed through 90 mph. In addition, the speed constraint requires that  $v \leq 111.8 \text{ m/s}$ . Since the calculated value is too high, the result will be the same as case 1-1. Thus, the minimum energy consumption for driving this distance will be at least 0.2597 kWh (1.1% of SOC consumed for a 24 kWh battery) when the speed is close to zero. However, if the maximum speed of the vehicle exceeds the constraint, according to the algorithm, we need to calculate  $v_{pp}$  since the calculated  $v_p > \sqrt{\frac{s*a}{(n+1)}}$ . The horizon line in Fig. 5 crosses the point at  $v=111.8$  and the other intersection is at  $v_{pp}=66.02$ . Thus, the vehicle is suggested to be driving at a speed of less than 66.02 m/s such that the energy consumption will be controlled to be less than 2.69 kWh (11.2% of SOC consumed for a 24 kWh battery) instead of the possible maximum 3.4 kWh.

TABLE II. CASE 1-1: RESULTS FOR NONSTOP TRIP WITHOUT REGENERATIVE ENERGY

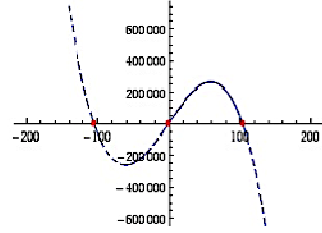
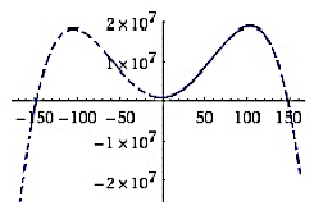
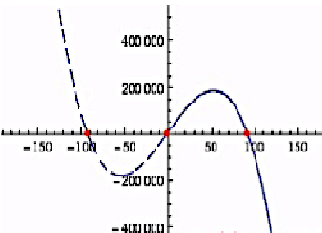
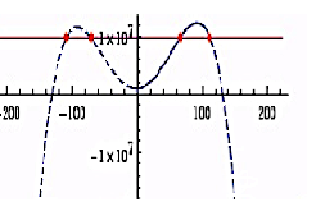
Results of the derivative for $n=0$ without regenerative energy:	Results of the energy consumption for $n=0$ without regenerative energy:
<p>Root plot:</p>  <p>Fig. 2. Root plot for case 1-1</p>	<p>Plots:</p>  <p>Fig. 3. Energy consumption plot for case 1-1</p>
<p>Solutions:</p> <p><math>v_1 = -104.439</math>  <math>v_2 = -1.180</math>  <math>v_3 = 103.577</math></p>	<p>Solutions:</p> <p><math>v_1 = -148.226</math> (ignore)  <math>v_2 = 147.838</math></p>

TABLE III. CASE 1-2: RESULTS FOR NONSTOP TRIP WITH REGENERATIVE ENERGY

Results of the derivative for $n=0$ with $\xi = 90\%$ :	Results of the energy consumption for $n=0$ with $\xi = 90\%$ :
 <p>Fig. 4. Root plot for case 1-2</p>	 <p>Fig. 5. Energy consumption plot for case 1-2</p>
<p>Solutions:</p> <p><math>v_1 = -92.313</math>  <math>v_2 = -1.507</math>  <math>v_3 = 91.777</math></p>	<p>Solutions:</p> <p><math>v_1 = -131.403</math> (ignore)  <math>v_2 = 131.628</math></p>

### B. Case 2: Computational Results for $n=20$ :

It can be seen from the plots in Table IV that the energy consumption at  $v = -0.211 \text{ m/s}$  should be a local minimum value, while the energy consumptions at  $v = -54.593 \text{ m/s}$  (ignore) and  $v = 52.761 \text{ m/s}$  should be the local maximum values. The speed constraint requires that  $v \leq 24.4 \text{ m/s}$ . Thus, the minimum energy consumption for driving this distance will be at least 0.2597 kWh (1.1% of SOC consumed for a 24 kWh battery) when the speed is close to zero. It can be noticed that, without any regenerative energy, the energy consumption will be a monotonically increasing function of velocity since the consumption of accelerating periods will take an increasingly significant proportion of the total consumption. Therefore, the following case studies will only consider the vehicle with regenerative braking function.

It can be seen from the plots in Table V that the energy consumption at  $v_2 = -1.135 \text{ m/s}$  should be a local minimum value, while the energy consumptions at  $v_1 = -23.607 \text{ m/s}$  (ignore) and  $v_3 = 22.699 \text{ m/s}$  should be local maximum values. If the speed range is from 0 up to 90 mph, the value at  $v = -1.135 \text{ m/s}$  may not be a global minimum since the

TABLE IV. CASE 2-1: RESULTS FOR 20 STOPS TRIP WITHOUT REGENERATIVE ENERGY

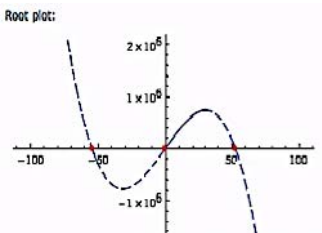
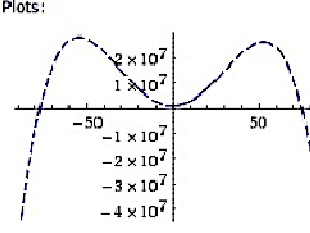
Results of the derivative for $n=20$ without regenerative energy:	Results of the energy consumption for $n=20$ without regenerative energy:
 <p>Fig. 6. Root plot for case 2-1</p>	 <p>Fig. 7. Energy consumption plot for case 2-1</p>
<p>Solutions:</p> $v_1 = -54.593$ ( <i>ignore</i> ) $v_2 = -0.211$ $v_3 = 52.761$	<p>Solutions:</p> $v_1 = -77.382$ ( <i>ignore</i> ) $v_2 = 75.097$

TABLE V. CASE 2-2: RESULTS FOR 20 STOPS TRIP WITH REGENERATIVE ENERGY

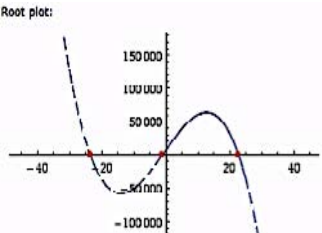
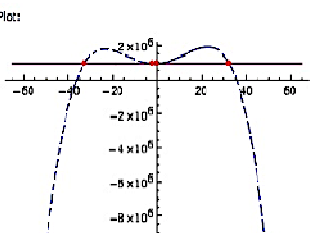
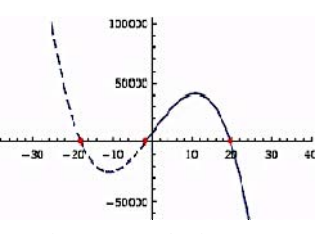
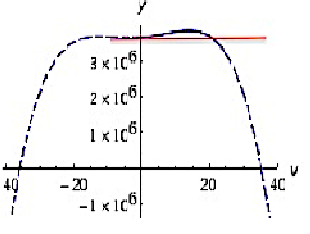
Results of the derivative for $n=20$ with regenerative energy, with $\xi = 90\%$ :	Results of the energy consumption for $n=20$ with regenerative energy, with $\xi = 90\%$ :
 <p>Fig. 8. Root plot for case 2-2</p>	 <p>Fig. 9. Energy consumption plot for case 2-2</p>
<p>Solutions:</p> $v_1 = -23.607$ ( <i>ignore</i> ) $v_2 = -1.135$ $v_3 = 22.699$	<p>Solutions:</p> $v_1 = -32.948$ ( <i>ignore</i> ) $v_2 = -2.271$ ( <i>ignore</i> ) $v_3 = 0$ $v_4 = 32.496$

TABLE VI. CASE 3: RESULTS FOR 20 STOPS TRIP WITH REGENERATIVE ENERGY AND INCLINATION OF GROUND

Results of the derivative for $n=20$ with $\xi = 90\%$ and $\theta=2^\circ$ :	Results of the energy consumption for $n=20$ with $\xi = 90\%$ and $\theta=2^\circ$ :
 <p>Fig. 10. Root plot for case 3</p>	 <p>Fig. 11. Energy consumption plot for case 3</p>
<p>Solutions:</p> $v_1 = -17.96$ ( <i>ignore</i> ) $v_2 = -1.716$ $v_3 = 19.675$	<p>Solutions:</p> $v_1 = -34.705$ ( <i>ignore</i> ) $v_2 = 34.327$

energy consumption will be decreasing after the speed reaches 22.699 m/s.

To verify whether the value with velocity close to 0 is the global minimum within the speed range, a horizon line is drawn in Fig. 9 which crosses the point at  $v = 0$ . The other intersection in the right half-plane is at  $v = 32.496$  m/s. Thus, when the speed is over 32.496 m/s, higher the speed lower will be the energy consumption. However, according to (18), the constraint for the speed is that it cannot exceed  $v = 24.4$  m/s when the stop times is  $n=20$ . Thus, in this case, the lesser the speed you drive, the lower will be the energy consumed by the vehicle. When the speed reaches 22.699 m/s, the energy consumption during the driving cycle will be the maximum value. By calculating the  $v_{pp}$  s.t.  $E(v_{pp}, 20, 0.9) = E(24.4, 20, 0.9)$ , we obtain  $v_{pp} = 20.745$ . Thus, the vehicle is suggested to be driving at a speed of less than 20.745 m/s such that the energy consumption will be controlled to be less than 0.528 kWh (2.1 % of SOC consumed for a 24 kWh battery) instead of reaching the possible maximum consumption.

Compared with the result of case 1-2, it indicates that the magnitude level of energy consumption with higher stop frequency is much lower. But the difference between case 1-1 and case 2-1 is not significant. The reason is that higher stop frequency will lead to lower speed that the vehicle can reach. The impact of the speed is greater with regenerative braking because it reduces the energy consumption during acceleration periods. However, this energy saving strategy with higher stop frequency is realized at the cost of consuming time. Considering the tradeoff between the two factors, the suggestions for the drivers in terms of the driving speed is provided in the optimization algorithm.

C. Case 3: Computational Results for  $n=20$  with inclination of ground of 2 degree:

It can be seen from the plots in Table VI that, the impact of the ground inclination is significant. Energy consumption at  $v_2 = -1.716$  m/s is a local minimum value, but no longer the global minimum value. The horizon line in Fig. 11 crosses the point at  $v=0$  and the other intersection is at  $v_p=17.956$  m/s. Thus, the vehicle is suggested to be driving at a speed of higher than 17.956 m/s such that the energy consumption will be controlled to be less than 1 kWh (4% of SOC consumed for a 24 kWh battery) instead of the possible maximum 1.05 kWh. The minimum energy consumption will be 0.9 kWh at the maximum speed which is 24.4 m/s in this case.

To compare the case 3 with case 2-2, we make the velocity in case 2-2 to be the same as 24.4 m/s, then the distance that the vehicle could drive to consume the same amount of energy as the minimum energy consumption in case 3 can be calculated as 7576 m. It indicates that energy consumption will be reduced if the extra distance to drive to avoid a  $2^\circ$  slope is less than 2576 m. Generally, it is cost-efficient if the extra distance to drive to avoid a  $\theta^\circ$  slope is less than

$$d=73830*\sin(\theta^\circ)+1006.07*\cos(\theta^\circ)-1006.07 \quad (21)$$

#### D. Summary

- The relationship between the average constant velocity and the total energy consumption can be demonstrated as a mound-shaped curve.
- The energy consumed during acceleration period cannot be neglected even for non-stop trips. If the technology permits, regenerative energy could help save a large proportion of energy, especially for frequent stop-and-go trips.
- The impact of ground inclination on the energy consumption is significant. The case study has indicated that driving a sort longer distance to avoid the slope may be more efficient.

#### VII. CONCLUSION

- In the proposed algorithm, optimal velocity for minimum energy usage under different driving profile is obtained. For those scenarios with no optimal result, practical guidance is provided for the drivers to control the energy consumption within a certain range.
- Such algorithm can be utilized in the vehicle embedded systems and provides suggestions to the drivers about the optimal driving speed based on the real-time driving condition and the preset driving cycle. The mathematical model and the optimization algorithm are also applicable if more factors are taken into consideration.

#### REFERENCES

- [1] Salmasi F.J., "Control Strategies for Hybrid Electric Vehicles: Evolution, Classification, Comparison, and Future Trends", IEEE Transactions on Vehicular Technology, vol. 56, 2007, pp 2393-2404
- [2] Wirasingha S.G. , Emadi A., "Classification and review of Control Strategies for Plug-In Hybrid Electric vehicles", IEEE Transactions on Vehicular Technology, vol. 60, 2011, pp 111-122
- [3] LeeH.andKooE.,SulS.,kimJ.,"Torquecontrolstrategyfora parallel- hybrid vehicle using fuzzy logic", IEEE Industry Applications Magazine, vol. 3, 1998, pp 1715-1720
- [4] Philips A.M. , Jankovic M. , Bailey K.,"Vehicle system controller design for a hybrid electric vehicle", IEEE International Conference on Control Applications, 2000, pp 297-302
- [5] Banvait H. , Sohel S. , Chen Y.,"A rule-based energy management strategy for plug-in hybrid electric vehicle (PHEV)", American Control Conference, 2009, pp 3938-3943
- [6] Delpar S. , Lauber J. , Guerra T.M. and Rimaux J.,"Control of a parallel hybrid powertrain: Optimal control",IEEE Transactions on Vehicular Technology, vol. 53, 2004, pp 872-881
- [7] Moura, S.J. , Fathy H.K. , Callaway D.S. , Stein J.L. , "A Stochastic Optimal Control Approach for Power Management in Plug-In Hybrid Electric Vehicles",IEEE Transactions on Control Systems Technology, vol. 19, 2010, pp 545-555
- [8] M. Moeini-Aghtaie, A. Abbaspur, M. Fotuhi-Firuzabad, and P. Dehghanian, "PHEV's Centralized/Decentralized Charging Control Mechanisms: Requirements and Impacts", *The 45<sup>th</sup> North American Power Symposium*, NAPS, 22-24 Sep. 2013, Kansas State, USA
- [9] Y. Jingyu, X. Guoqing, X. Yangsheng, and X. Benliang, "Battery state-of-charge estimation based on H  $\infty$  filter for hybrid electric vehicle," presented at the Control, Automation, Robotics and Vision, 2008. ICARCV 2008. 10th International Conference on, 2008
- [10] P. Dehghanian, S. H. Hosseini, M. Moeini-Aghtaie, and S. Arabali, "Optimal Siting of DG Units in Power Systems form a Probabilistic Multi-Objective Optimization Perspective", *International Journal of Electrical Power and Energy Systems*, vol.51, pp.14-26, October 2013
- [11] Cheng Wang, "Prediction of battery-SOC of pure electric vehicle," Computer Science & Education (ICCSE), 2012 7th International Conference on , vol., no., pp.466,469, 14-17 July 2012
- [12] Xudong Liu; Qingwu Fan; Kun Zheng; Jianmin Duan; Yu Wang, "Constant SOC control of a series Hybrid Electric Vehicle with long driving range," Information and Automation (ICIA), 2010 IEEE International Conference on , vol., no., pp.1603,1608, 20-23 June 2010
- [13] Zhu Daowei; Xie Hui, "Control strategy optimization of the hybrid electric bus based on remote self-learning driving cycles," Vehicle Power and Propulsion Conference, 2008. VPPC '08. IEEE , vol., no., pp.1,5, 3-5 Sept. 2008
- [14] Weimin Li; Guoqing Xu; Zhancheng Wang; Yangsheng Xu, "Dynamic energy management for hybrid electric vehicle based on adaptive dynamic programming," Industrial Technology, 2008. ICIT 2008. IEEE International Conference on , vol., no., pp.1, 6, 21-24 April 2008
- [15] Lot, R.; Evangelou, S.A., "Green driving optimization of a series hybrid electric vehicle," Decision and Control (CDC), 2013 IEEE 52nd Annual Conference on , vol., no., pp.2200,2207, 10-13 Dec. 2013
- [16] M. Miyatake and H. Ko, "Optimization of train speed profile for minimum energy consumption," IEEE Transactions on Electrical and Electronic Engineering, vol. 5, no. 3, pp. 263–269, 2010.
- [17] Mk Rizzo, D.M.; Parker, G.G., "State of charge optimization for military hybrid vehicle microgrids," Innovative Smart Grid Technologies (ISGT), 2013 IEEE PES , vol., no., pp.1,6, 24-27 Feb. 2013
- [18] Mk Yi Zhang; Heping Liu; Qiang Guo, "Varying-Domain Optimal Management Strategy for Parallel Hybrid Electric Vehicles," Vehicular Technology, IEEE Transactions on , vol.63, no.2, pp.603,616, Feb. 2014
- [19] Pavlat, J.W.; Diller, R.W., "An energy management system to improve electric vehicle range and performance," Aerospace and Electronic Systems Magazine, IEEE , vol.8, no.6, pp.3,5, June 1993
- [20] Caillard, P.; Gillon, F.; Hecquet, M.; Janiaud, N., "Influence of machine control strategy on electric vehicle range," Ecological Vehicles and Renewable Energies (EVER), 2013 8th International Conference and Exhibition on , vol., no., pp.1,6, 27-30 March 2013
- [21] Yuhe Zhang; Wenjia Wang; Kobayashi, Y.; Shirai, K., "Remaining driving range estimation of electric vehicle," Electric Vehicle Conference (IEVC), 2012 IEEE International , vol., no., pp.1,7, 4-8 March 2012
- [22] Grossard, M.; Kachroudi, S.; Abroug, N., "An optimal energy-based approach for driving guidance of full Electric Vehicles," Industrial Electronics (ISIE), 2012 IEEE International Symposium on , vol., no., pp.1708,1713, 28-31 May 2012
- [23] Rui Wang; Lukic, S.M., "Review of driving conditions prediction and driving style recognition based control algorithms for hybrid electric vehicles," Vehicle Power and Propulsion Conference (VPPC), 2011 IEEE , vol., no., pp.1,7, 6-9 Sept. 2011
- [24] Sado, H.; Sakai, S.; Hori, Y., "Road condition estimation for traction control in electric vehicle," Industrial Electronics, 1999. ISIE '99. Proceedings of the IEEE International Symposium on , vol.2, no., pp.973,978 vol.2, 1999
- [25] L. Guzzella and A. Sciarretta, *Vehicle propulsion systems*: Springer, 2007
- [26] Jed, H.A., "An electric vehicle model and a driving cycle for mail delivery use," Power Electronics and Applications (EPE 2011), Proceedings of the 2011-14th European Conference on Aug. 30 2011- Sept. 1 2011, Birmingham, England
- [27] S. Shahidinejad, E. Bibeau, and S. Filizadeh, Statistical Development of a Duty Cycle for Plug-in Vehicles in a North American Urban Setting Using Fleet Information, IEEE Transactions On Vehicular Technology, Vol. 59, No. 8, October 2010
- [28] J. Y. Wong, *Theory of Ground Vehicles*. New York: Wiley-Interscience, 2001
- [29] Nissan Leaf Review. [Online] <http://www.plugincars.com>